Journal of Functional Analysis 274 (2018) 797–825

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

Type, cotype and twisted sums induced by complex interpolation ☆

Willian Hans Goes Corrêa

Departamento de Matemática, Instituto de Matemática e Estatística, Universidade de São Paulo, Rua do Matão 1010, 05508-090 São Paulo SP, Brazil

A R T I C L E   I N F O

Article history:
Received 6 March 2017
Accepted 17 November 2017
Available online 2 December 2017
Communicated by G. Schechtman

Keywords:
Twisted sums
Complex interpolation
Banach spaces

A B S T R A C T

This paper deals with extensions or twisted sums of Banach spaces that come induced by complex interpolation and the relation between the type and cotype of the spaces in the interpolation scale and the nontriviality and singularity of the induced extension. The results are presented in the context of interpolation of families of Banach spaces, and are applied to the study of submodules of Schatten classes. We also obtain nontrivial extensions of spaces without the CAP which also fail the CAP.

© 2017 Elsevier Inc. All rights reserved.

1. Introduction

A twisted sum of Banach spaces Y and Z is a short exact sequence

$$0 \longrightarrow Y \xrightarrow{i} X \xrightarrow{q} Z \longrightarrow 0,$$

where X is a quasi-Banach space and the arrows are bounded linear maps.

☆ The present work was produced with support from CNPq, National Council for Scientific and Technological Development – Brazil, grant 140413/2016-2, and CAPES, Coordenação de Aperfeiçoamento de Pessoal de Nível Superior, grant 1328372.

E-mail address: willhans@ime.usp.br.

https://doi.org/10.1016/j.jfa.2017.11.010
0022-1236/© 2017 Elsevier Inc. All rights reserved.

The theory of twisted sums of Banach spaces has been successfully used in the study of the so called 3-space properties. Given a property P of Banach spaces, P is said to be a 3-space property (3SP) if for every Banach space X having a closed subspace with P and respective quotient also with P, X has P. Simple examples of 3SP are separability, having finite dimension and reflexivity.

To see how twisted sums and 3SP are related, consider the following problem of Palais: is being isomorphic to a Hilbert space a 3SP? Finding a counterexample to this problem corresponds to finding a twisted sum

$$0 \longrightarrow H_1 \longrightarrow X \longrightarrow H_2 \longrightarrow 0,$$

where H_1 and H_2 are a Hilbert spaces and H_1 is not complemented in X (in this case, X is automatically isomorphic to a Banach space).

In [21], Enflo, Lindenstrauss and Pisier show that there is such counterexample. Kalton and Peck gave a solution some years later ([28]) using nonlinear maps called quasi-linear. They obtained a twisted sum:

$$0 \longrightarrow \ell_2 \longrightarrow Z_2 \longrightarrow \ell_2 \longrightarrow 0 \tag{1.1}$$

in which the copy of ℓ_2 in Z_2 is not complemented.

The Kalton–Peck space Z_2 also appears in a construction due to Rochberg and Weiss which is possible every time we have a compatible pair (X_0, X_1) in the sense of interpolation (see [35]). Given (X_0, X_1), for each $\theta \in (0, 1)$ we have a twisted sum of X_θ with itself, denoted dX_θ. In this context the Kalton–Peck space comes induced by the interpolation scale (ℓ_∞, ℓ_1) at $\theta = \frac{1}{2}$.

Given two Banach spaces Y and Z, we always have a trivial twisted sum by means of the direct sum $Y \oplus Z$ with the obvious inclusion and quotient map. So the first objective in the theory is obtaining nontrivial twisted sums, i.e., twisted sums in which the subspace is not complemented in the middle space.

In [26], under the assumption of super-reflexivity, Kalton shows that the twisted sum induced by a compatible pair of Köthe function spaces (X_0, X_1) is boundedly trivial (a subclass of trivial twisted sums) exactly when $X_0 = X_1$. One of the results of a preprint of the author with Castillo, Ferenczi and González ([10]) is that, under the same assumptions, the twisted sum induced by a compatible pair of Köthe function spaces (X_0, X_1) at θ is trivial precisely when X_1 is a weighted version of X_0. The question of which interpolation scales generate trivial twisted sums is still open in the general scenario.

Banach's Hyperplane Problem asks if Banach spaces are always isomorphic to its hyperplanes. In [24], Gowers answered this problem in the negative. The Kalton–Peck space is naturally isomorphic to its subspaces of codimension 2, but it is still unknown if it is isomorphic to its hyperplanes. Since it appears more naturally than Gower's construction, it would be interesting to know if it also answers in the negative the Hyperplane Problem.

Download English Version:

https://daneshyari.com/en/article/8896782

Download Persian Version:

https://daneshyari.com/article/8896782

Daneshyari.com