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CIRCLES IN THE SPECTRUM AND THE GEOMETRY OF ORBITS: A NUMERICAL RANGES APPROACH

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Abstract. We prove that a bounded linear Hilbert space operator has the unit circle in its essential approximate point spectrum if and only if it admits an orbit satisfying certain orthogonality and almost-orthogonality relations. This result is obtained via the study of numerical ranges of operator tuples where several new results are also obtained. As consequences of our numerical ranges approach, we derive in particular wide generalizations of Arveson's theorem as well as show that the weak convergence of operator powers implies the uniform convergence of their compressions on an infinite-dimensional subspace.

1. Introduction

It is well-known that in the study of invariant subspaces of a bounded linear operator T on a Hilbert space, the presence of the unit circle $\mathbb{T}$ in the spectrum $\sigma(T)$ of T plays a special role. According to one of the strongest results in this direction due to Brown, Chevreau and Pearcy [7], see also [4] and [20, p.156 -157], if T is a Hilbert space contraction having the unit circle in its spectrum, then T has a non-trivial invariant subspace. The statement was extended to Banach spaces and to polynomially bounded operators, [1]. For this and related statements one may also consult the recent survey [5], and the books [18, Chapter 5] and [8]. However the spectral condition $\sigma(T) \supset \mathbb{T}$ appeared to be again crucial. Thus, it is of substantial interest to clarify its interplay with the behavior of orbits of T.

That issue has not received adequate attention in the literature. Curiously enough, known results on the implications of the circle structure of the spectrum for the geometry of orbits have been noted in an area somewhat distant from classical operator theory. A long time ago, Arveson proved in [3] that the spectrum of a unitary operator T on H is precisely the unit circle $\mathbb{T}$ if and only if for every $n \in \mathbb{N}$ there exists a nonzero $x \in H$ such that the elements $x, Tx, ..., T^n x$ are mutually orthogonal. His motivation

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