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Norm of the Hilbert matrix on Bergman spaces

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Abstract. It is well known that the Hilbert matrix operator H is a bounded operator from the Bergman space A^p into A^p if and only if $2 < p < \infty$. In [**5**] it was shown that the norm of the Hilbert matrix operator H on the Bergman space A^p is equal to $\frac{\pi}{\sin\frac{2\pi}{p}}$, when $4 \le p < \infty$, and it was also conjectured that

$$\|H\|_{A^p \to A^p} = \frac{\pi}{\sin\frac{2\pi}{p}},$$

when $2 < p < 4$. In this paper we prove this conjecture.

1. Introduction

The first upper bound for the norm of the Hilbert matrix operator H on the Bergman space A^p, when $2 < p < \infty$, has been obtained by Diamantopoulos in [**3**]. It was also observed there, that H is not bounded on A^2. In the case $4 \le p < \infty$, by using representation of the Hilbert matrix operator H as an average of certain weighted composition operators, he obtained the estimate

$$\|H\|_{A^p \to A^p} \le \frac{\pi}{\sin\frac{2\pi}{p}}.$$

He has also obtained a less precise estimate for the norm of the Hilbert matrix operator H on A^p, when $2 < p < 4$. These results were improved and extended in [**5**], by Dostanić, Jevtić and Vukotić, where the exact value of the norm of H on A^p, when $4 \le p < \infty$, was found. They proved that

$$\|H\|_{A^p \to A^p} = \frac{\pi}{\sin\frac{2\pi}{p}},$$

when $4 \le p < \infty$. In [**5**] it was conjectured that this also holds for $2 < p < 4$. However, it seems that the case $2 < p < 4$ is beyond the scope of the previous methods used to estimate the exact norm of the Hilbert matrix operator and that a novel approach is required.

In this paper we prove the conjecture from [**5**], settling the question about the norm of the Hilbert matrix operator on Bergman spaces entirely. We base our method on a new way to use monotonicity of integral means, and reduce the conjecture about norm to some inequalities for Beta functions, that also seem to be new.

First, we give a quick review of the relevant background (also see [**7, 11, 13**]) and give an overview of our main results.

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