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Decay estimates and Strichartz estimates of fourth-order Schrödinger operator

Hongliang Feng [a], Avy Soffer [a,1], Xiaohua Yao [b,*]

[a] School of Mathematics and Statistics, Central China Normal University, Wuhan, 430079, PR China

[b] Department of Mathematics and Hubei Province Key Laboratory of Mathematical Physics, Central China Normal University, Wuhan, 430079, PR China

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A B S T R A C T

We study time decay estimates of the fourth-order Schrödinger operator $H = (-\Delta)^2 + V(x)$ in $\mathbb{R}^d$ for $d = 3$ and $d \geq 5$. We analyze the low energy and high energy behaviour of resolvent $R(H; z)$, and then derive the Jensen–Kato dispersion decay estimate and local decay estimate for $e^{-itH}P_{ac}$ under suitable spectrum assumptions of H. Based on Jensen–Kato type decay estimate and local decay estimate, we obtain the $L^1 \to L^\infty$ estimate of $e^{-itH}P_{ac}$ in 3-dimension by Ginibre argument, and also establish the endpoint global Strichartz estimates of $e^{-itH}P_{ac}$ for $d \geq 5$. Furthermore, using the local decay estimate and the Georgescu–Larenas–Soffer conjugate operator method, we prove the Jensen–Kato type decay estimates for some functions of H.

* Corresponding author.
E-mail addresses: fenghongliangccnu@163.com (H. Feng), soffer@math.rutgers.edu (A. Soffer), yaoxiaohua@mail.ccnu.edu.cn (X. Yao).

[1] On leave from Rutgers University.

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Contents

1. Introduction

In this paper we consider the time decay estimates of the operator

$$H = H_0 + V(x), \ H_0 = (-\Delta)^2$$

in $L^2(\mathbb{R}^d)$ for $d = 3$ and $d \geq 5$, where $V(x)$ is a real valued function as a multiplication operator. In the sequel, we assume that $V(x) = O(|x|^{-\beta})$ for large $|x|$ with some $\beta > 0$ (the specific β will be given in conclusions below).

It is well known that the fourth-order Schrödinger equation was introduced by Karpman [32,33] and Karpman and Shagalov [31] to take into account the role of small fourth-order dispersion terms in the propagation of intense laser beams in a bulk medium with Kerr nonlinearity. The nonlinear beam equation, or fourth-order wave equation has been involved in the study of plate and beams, see e.g. Love [45], in the study of interaction of water waves, see Bretherton [8], and in the study of the motion of a suspension bridge, see Lazer and MacKenna [41] and MacKenna and Walter [46,47]. Recently, these fourth-order equations were considered in mathematics by many authors. For example, Levandosky and Strauss had considered the stability and instability of fourth-order solitary waves [42], the time decay estimates for fourth-order wave equations [43] and [44]. Moreover, the well-posedness and scattering problems of nonlinear fourth-order Schrödinger equation have been further studied by many authors now, see e.g. Miao, Xu and Zhao [48,49], Pausader [55,56], C. Hao, L. Hsiao and B. Wang [19,20], Ruzhansky, B. Wang and H. Zhang [59], Segata [62,63] and references therein.

In the studies of linear or nonlinear dispersive equations, one is faced with the need to quantitatively estimate the time decay of the solution in different kinds of norms.