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I-factorial quantum torsors and Heisenberg algebras of quantized universal enveloping type

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A B S T R A C T

We introduce a notion of *I-factorial quantum torsor*, which consists of an integrable ergodic action of a locally compact quantum group on a type I-factor such that also the crossed product is a type I-factor. We show that any such I-factorial quantum torsor is at the same time a I-factorial quantum torsor for the dual locally compact quantum group, in such a way that the construction is involutive. As a motivating example, we show that quantized compact semisimple Lie groups, when amplified via a crossed product construction with the function algebra on the associated weight lattice, admit I-factorial quantum torsors, and give an explicit realization of the dual quantum torsor in terms of a deformed Heisenberg algebra for the Borel part of a quantized universal enveloping algebra.

0. Introduction

The theory of *locally compact quantum groups* [15,16] provides a vast generalization of the classical theory of locally compact groups. Being steeped in the theory of von

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Neumann algebras and Tomita–Takesaki theory, it is the proper setting in which to study quantum symmetries such as arising for example from subfactor theory [8,7,26]. One of the main attributes of the theory is the existence of a *generalized Pontryagin duality theory*, allowing for a uniform treatment of many classical group theoretical results and constructions.

In this article, we will be concerned with the theory of *Galois objects*. These structures first arose in the setting of Hopf algebras as the proper generalization of the notion of *torsor*, see [24] for an overview. A theory of Galois objects in the analytic framework of locally compact quantum groups was developed in [3]. They can be defined as von Neumann algebras with an ergodic and integrable action of a locally compact quantum group such that the crossed product von Neumann algebra is a type I-factor. In this paper we will study Galois objects which are *themselves* type I-factors. These Galois objects will be called *I-factorial Galois objects* or *I-factorial quantum torsors*. Our main result shows that the natural adjoint action of the dual quantum group is again a I-factorial Galois object, and that moreover this construction is involutive.

This situation is not as uncommon as it may seem on first sight. In the setting of finite-dimensional Hopf algebras, such structures were studied in [1, Section 5]. In the analytic setting, there is a canonical class of examples whereby the *Heisenberg double* of a locally compact quantum group [27], which is the crossed product von Neumann algebra of the locally compact quantum group by the translation action on itself, becomes a I-factorial quantum torsor for the Cartesian product of the quantum group with its dual. However, our main focus in this article will be on a class of I-factorial quantum torsors which are of Heisenberg type in the *complex analytic* setting, but not in the *real analytic* setting: they arise as a (deformed) Heisenberg double of a complex Hopf algebra, and come equipped with a $*$-structure swapping the holomorphic and the anti-holomorphic parts. This purely algebraic data contains however insufficient spectral information to integrate this structure to the locally compact quantum setting, and a more hands-on approach needs to be taken for this, using the general theory developed in the first part of this paper.

The complex Hopf algebra that we are concerned with is the quantized universal enveloping algebra $U_q(\mathfrak{b})$ of a Borel subalgebra $\mathfrak{b}$ of a semisimple complex Lie algebra $\mathfrak{g}$. Indeed, this Hopf algebra has a natural skew self-pairing, giving rise to a *Heisenberg double* and a *Drinfeld double*, the latter being an amplification of the *quantized universal enveloping algebra* $U_q(\mathfrak{g})$. The key observation, allowing to connect the algebraic with the analytic framework, will be a variation on the fact that the Heisenberg algebra of the nilpotent part of the quantized Borel algebra can be realized as the function algebra on the (big open) *quantum Schubert cell* associated to $\mathfrak{g}$, see [11, Section 10] and the more recent works [9,13,5].

The precise contents of this paper are as follows.

In the *first four sections*, we deal with the general analytic theory. In Section 1, we recall the notion of a locally compact quantum group [16]. In Section 2, we introduce the notion of *Galois object* for a locally compact quantum group [3], and recall some of the