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Free transport for interpolated free group factors

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ABSTRACT

In this article, we study a form of free transport for the interpolated free group factors, extending the work of Guionnet and Shlyakhtenko for the usual free group factors [11]. Our model for the interpolated free group factors comes from a canonical finite von Neumann algebra $\mathcal{M}(\Gamma, \mu)$ associated to a finite, connected, weighted graph (Γ, V, E, μ) [12,13]. With this model, we use an operator-valued version of Voiculescu's free difference quotient introduced in [13] to state a Schwinger–Dyson equation which is valid for the generators of $\mathcal{M}(\Gamma, \mu)$. We construct free transport for appropriate perturbations of this equation. Also, $\mathcal{M}(\Gamma, \mu)$ can be constructed using the machinery of Shlyakhtenko's operator-valued semicircular systems [24].

0. Introduction

The interpolated free group factors $L(\mathbb{F}_t)$ for $t \in (1, \infty]$ were discovered and developed independently by Dykema [5] and Rădulescu [22]. They satisfy the following properties:

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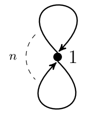
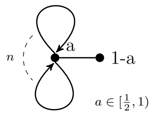
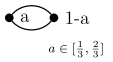
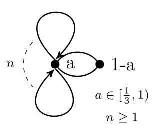
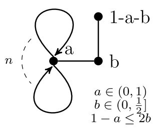
Γ	$\mathcal{M}(\Gamma, \mu)$
	$L(\mathbb{F}_n)$
	$L(\mathbb{F}_t)$ $t = (n-4)a^2 + 4a$
	$L(\mathbb{F}_t)$ $t = 6(a - a^2)$
	$L(\mathbb{F}_t)$ $t = (n-6)a^2 + 6a$
	$L(\mathbb{F}_t)$ $t = (n-2)a^2 - 4b^2 - 2ab + 2a + 3b$

Fig. 1. von Neumann algebras corresponding to simple graphs.

- $L(\mathbb{F}_t) * L(\mathbb{F}_s) = L(\mathbb{F}_{s+t})$.
- $pL(\mathbb{F}_t)p = L(\mathbb{F}_r)$ where $r = 1 + \frac{t-1}{\mathrm{tr}(p)^2}$ and p is a nonzero projection in $L(\mathbb{F}_t)$.
- If $t \in \mathbb{N} \cup \{\infty\}$ then $L(\mathbb{F}_t)$ is the usual free group factor on t generators.

For non-integer t, these share many of the same properties of their integer counterparts. Namely, they are non-Γ, strongly solid II_1 factors. In this paper, we demonstrate another similarity: the existence of free transport.

For our purposes, the most convenient way to describe an interpolated free group factor is via a weighted graph. Specifically, we consider a finite, connected, undirected, weighted graph (Γ, V, E, μ) with vertex set V, edge set E, and weighting $\mu : V \to (0, 1]$ satisfying $\sum_{v \in V} \mu(v) = 1$. One can associate to this data a C*-algebra $\mathcal{S}(\Gamma, \mu)$ and a von Neumann algebra $\mathcal{M}(\Gamma, \mu)$. A simple non-degeneracy condition on the weighting determines whether $\mathcal{M}(\Gamma, \mu)$ is a factor, and when $\mathcal{S}(\Gamma, \mu)$ is simple with unique trace. If $\mathcal{M}(\Gamma, \mu)$ is a factor, then it is necessarily isomorphic to $L(\mathbb{F}_t)$ where

$$t = 1 - \sum_{v \in V} \mu(v)^2 + \sum_{v \in V} \mu(v) \sum_{w \sim v} n_{v,w} \mu(w).$$

See Equation (1) and the discussion immediately preceding it. In particular, if Γ consists of a single vertex with n-loops, $\mathcal{M}(\Gamma, \mu) \cong L(\mathbb{F}_n)$, and $\mathcal{S}(\Gamma, \mu)$ is the C*-algebra generated by a free semicircular system (*cf.* Fig. 1). The algebras $\mathcal{M}(\Gamma, \mu)$ were initially studied in [8] in determining the isomorphism classes of von Neumann algebras arising from planar

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