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ACCEPTED MANUSCRIPT

Character sums of Lee and Weintraub

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Abstract

We prove two conjectures of Lee and Weintraub and one conjecture of Ibukiyama and Kaneko about character sums arising as fixed point contributions in the Atiyah-Singer holomorphic Lefshetz formula applied to finite group actions on the space of certain Siegel cusp forms.

Keywords: Character sums, Exponential sums

1. Introduction

Fix an odd prime p and let ψ denote the Legendre symbol mod p: $\psi(a) = \left(\frac{a}{p}\right)$. We put $\zeta = exp(2\pi i/p)$. Let $P(x), Q(x), R(x)$ be polynomials with integer coefficients. Following Lee-Weintraub[1], we define the Lee-Weintraub sum $S[P, Q, R]$ by

$$S[P,Q,R] = -\sum_{k,j(p)}{}' \frac{\psi(k)}{(\zeta^{kP(j)}-1)(\zeta^{kQ(j)}-1)(\zeta^{kR(j)}-1)},$$

where k and j each run over a complete residue system modulo p, and the prime on the summation sign means that the meaningless terms are to be excluded.

These sums are of interest for two reasons. Firstly, they are related to special values of L-functions which are then related to the theory of modular forms, so they are related to many fundamental objects of number theory and therefore deserve to be studied. Secondly, the polynomials P, Q, R may be of degree greater than 1, which makes these sums exotic. Accordingly, the treatment of these sums requires new techniques.

Lee-Weintraub encountered these sums while studying the action of the symplectic group $Sp(4, \mathbb{F}_p)$ on the space of Siegel cusp forms of degree 2 belonging to the principal congruence subgroup $\Gamma(p)$ by using the holomorphic Lefshetz fixed-point theorem of Atiyah-Singer, where traces of this action gave rise to certain Lee-Weintraub sums. Arakawa-Hashimoto on the other hand studied this action by using the Selberg trace formula, where certain traces of this action were expressed by special values of certain L-functions. In [2], we expressed the special values of these L-functions by finite sums of products of Bernoulli

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