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Abstract

Let p be a prime number, $q = p^r$ with $r \geq 2$ and $P \in \mathbb{F}_q[X]$. In this paper, we first estimate the number of $x \in \mathbb{F}_q$ such that $P(x)$ has prescribed digits (in the sense of Dartyge and Sárközy). In particular, for a given proportion < 0.5 of prescribed digits, we show that this number is asymptotically as expected. Then, we obtain similar results when x is allowed to run only in the set of generators (primitive elements) of $\mathbb{F}_q^*$. In the case of special interest where P is a monomial of degree 2, our estimate for the number of $x \in \mathbb{F}_q$ such that $P(x)$ has prescribed digits is sharper than the estimate following from the Weil bound. We will need to study exponential sums of independent interest such as multiplicative character sums over affine subspaces and additive character sums with generator arguments.

Keywords: finite fields, prescribed digits, character sums, squares, primitive elements
2010 MSC: 11A63, 11T23, 11B83

1. Introduction

1.1. Motivation

Let $g \geq 2$ be an integer. Every integer $n \in \mathbb{N}$ can be written uniquely in base g:

$$n = \sum_{j=0}^{r} c_j g^j \tag{1}$$

where the digits c_j belong to $\{0, \ldots, g-1\}$ and $c_r \geq 1$. The study of the connection between the arithmetic properties of n and the properties of its digits in a given basis produces a lot of interesting and difficult questions. Many results

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