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Tame kernels of cubic and sextic fields *

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Abstract Let K be a non-Galois cubic field, and let F denote the normal closure of $K/\mathbb{Q}$ or a sextic cyclic field. In this paper, we establish some relations between the p-rank of $K_2\mathcal{O}_K$ (resp. $K_2\mathcal{O}_F$) and the p-rank of the ideal class groups of some subfields of $K(\zeta_p)$ (resp. $F(\zeta_p)$). In the case of $p = 3$, we obtain estimates for the p-ranks of tame kernels $K_2\mathcal{O}_K$ (resp. $K_2\mathcal{O}_F$).

Key words Cubic fields, Sextic fields, Class groups, Tame kernels

MSC Primary 11R70, Secondary 11R16, 11R20, 11R29, 19F15

1 Introduction

Let F be a number field and $\mathcal{O}_F$ the ring of integers in F. It is well known that the Milnor K-group $K_2\mathcal{O}_F$ is the same as the tame kernel of F. For any prime p, it is interesting to get the value of the p-rank $K_2\mathcal{O}_F$. Let ζ_p be a primitive root of unity of degree p. If $\zeta_p \in F$, one can deduce the p-rank formula of $K_2\mathcal{O}_F$ from Tate's theorem (cf. [6] and [17]). For a quadratic number field, $K_2\mathcal{O}_F$ has been intensively studied, and some significant results about the 2-Sylow subgroup of $K_2\mathcal{O}_F$ have been obtained (cf. [9], [10], [11], [12], [13]). Moreover, one can determine explicitly the structure of the 2-Sylow subgroup of $K_2\mathcal{O}_F$ for some quadratic number fields by Qin's results. If $\zeta_p \notin F$, then we have the following short exact sequence

$$0 \to (\mu_p \otimes Cl(\mathcal{O}_E[1/p]))^G \to K_2\mathcal{O}_F/p \to \oplus_{\mathfrak{p}\in S'}\mu_p \to 0, \qquad (*)$$

where μ_p is the group of p-th root of unity, $E = F(\zeta_p)$, $G = \mathrm{Gal}(E/F)$ acts on $\mu_p \otimes Cl(\mathcal{O}_E[1/p])$ by the formula

$$(\zeta \otimes x)^\sigma = \zeta^\sigma \otimes x^\sigma, \quad \text{for } \zeta \in \mu_p, \ \sigma \in G, \quad x \in Cl(\mathcal{O}_E[1/p]),$$

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