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### THREE RAMANUJAN CONTINUED FRACTIONS WITH MODULARITY

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ABSTRACT. We study three Ramanujan continued fractions  $c(\tau)$ ,  $W(\tau)$  and  $T(\tau)$ . In fact,  $c(\tau)$  and  $W(\tau)$  are modular functions of level 16, and  $T(\tau)$  is a modular function of level 32. We first prove that the values of  $c(\tau)$  and  $W(\tau)$  can generate the ray class field modulo 4 over an imaginary quadratic field  $K$ . We also prove that  $2/(1 - c(\tau))$ ,  $1/W(\tau)$ ,  $T(\tau) + 1/T(\tau)$  are algebraic integers for any imaginary quadratic quantity  $\tau$ . Furthermore, we find the modular equations of  $c(\tau)$ ,  $T(\tau)$  and  $W(\tau)$  for any level, and we show that  $c(\tau)$  and  $W(\tau)$  satisfy the Kronecker's congruence. We can express the value  $c(r\tau)$  (respectively,  $T(r\tau)$ ,  $W(r\tau)$ ) in terms of radicals for any positive rational number  $r$  when the value  $c(\tau)$  (respectively,  $T(\tau)$ ,  $W(\tau)$ ) can be written as radicals.

#### 1. INTRODUCTION

The *Rogers-Ramanujan continued fraction*  $r(\tau)$  is a continued fraction which is written in two letters by Ramanujan to Hardy [16, pp. xxvii, xxviii], and it is defined by

$$r(\tau) = \frac{q^{1/5}}{1 + \frac{q}{1 + \frac{q^2}{1 + \frac{q^3}{1 + \dots}}}} \quad (q := e^{2\pi i\tau}).$$

Ramanujan's letters contain several theorems including the evaluation of  $r(\tau)$  and the relations between  $r(\tau)$  and  $r(n\tau)$  ( $n = 2, \dots, 5$ ). These amazing results are recently proved by the theory of modular forms [3, 6, 7] because  $r(\tau)$  is a generator of the field of modular functions on  $\Gamma(5)$ .

There are several works which are motivated by Ramanujan's work. Among them, we introduce the following three continued fractions  $c(\tau)$ ,  $T(\tau)$  and  $W(\tau)$ :

$$c(\tau) = \frac{1}{1 + \frac{2q}{1 - q^2 + \frac{q^2(1 + q^2)^2}{1 - q^6 + \frac{q^4(1 + q^4)^2}{1 - q^{10} + \dots}}}},$$

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