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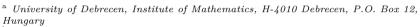
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On the equation $A!B! = C!^{*}$





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ABSTRACT

We consider the equation in the title in positive integers A, B, C. We give an explicit upper bound for C in terms of the difference k := B - A. Further, we show that for $k \le 10^6$ this equation has only one (long known) non-trivial solution, given by 6!7! = 10!.

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1. Introduction and the main result

The question of finding all products of factorials yielding a factorial is a long standing problem, studied by many authors. Here we only mention a few related results; for a survey of the topic see e.g. Guy [8], section B23. Consider the equation

$$n! = \prod_{i=1}^{r} a_i! \tag{1}$$

with $r \geq 2$ in positive integers n, a_1, \ldots, a_r , with $a_1 \geq \cdots \geq a_r > 1$. Observe that this equation has infinitely many solutions given by

$$n = a_2! \dots a_r!$$
, $a_1 = n - 1$, with a_2, \dots, a_r arbitrary.

For example, we have 6! = 5!3! or 12! = 11!3!2!. Such solutions are called trivial. Obviously, equation (1) has infinitely many trivial solutions. On the other hand, according to a conjecture of Surányi, the only non-trivial solution to (1) with r = 2 is 10! = 7!6!, while a conjecture of Hickerson predicts that the only non-trivial solutions to (1) are given by 9! = 7!3!3!2!, 10! = 7!6! = 7!5!3!, 16! = 14!5!2! (see e.g. Erdős [4], pp. 27–28). These conjectures have been checked for $n \leq 10^6$ by Caldwell [3]. Erdős [4] (see Theorem 2) proved that writing P(m) for the largest prime factor of the positive integer m (with the convention P(1) = 1), the assertion

$$P(n(n+1)) > 4\log n \tag{2}$$

would imply that equation (1) has only finitely many non-trivial solutions – however, (2) is far from being established. (See also [7], p. 70.) Luca [10] proved that assuming the *abc*-conjecture, (1) has only finitely may solutions. This result (beside obtaining other related theorems) has been made more explicit by Luca, Saradha and Shorey [11].

We also mention that after multiplying both sides of (1) by n!, we get an equation of the form

$$n! \prod_{i=1}^{r} a_i! = y^2.$$

This equation also attracted a lot of attention. For related results, here we only mention a classical paper of Erdős and Graham [6] together with the recent paper of Luca, Saradha and Shorey [11], and the references there.

In this paper we consider the case r=2, and rewrite equation (1) as

$$A!B! = C! \tag{3}$$

with positive integers A, B, C satisfying $C \ge B \ge A > 1$.

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