ARTICLE IN PRESS YJNTH:5928

Journal of Number Theory ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Number Theory

www.elsevier.com/locate/jnt

A conjectural characterization for $\mathbb{F}_q(t)$-linear relations between multizeta values

George Todd

Department of Mathematics, Bailey Hall 202, Union College, Schenectady, NY 12308, United States

ARTICLE INFO

Article history:
Received 9 February 2016
Received in revised form 23 September 2017
Accepted 27 September 2017
Available online xxxx
Communicated by S.J. Miller

Keywords:
Multizeta values
Multizeta function
Function fields

ABSTRACT

We provide a conjectural characterization for $\mathbb{F}_q(t)$-linear relations between multizeta values and for the dimensions of their fixed-weight $\mathbb{F}_q(t)$ span, as well as many new parameterized families of relations. These two conjectures provide the function field analog of the conjectures provide by Zagier and others. In contrast to the classical case which uses regularized stuffle and shuffle products to produce relations, we will posit that all relations in the function field setting can be generated from a single relation in weight q from Thakur's stuffle product. We also prove many of the currently existing relations in the literature in this setting.

1. Introduction

There exist two products on the $\mathbb{Q}$-span of classical multizeta values, the shuffle product and the stuffle product, from which $\mathbb{Q}$-linear relations can be derived. It has been conjectured that the regularization of this process accounts for all such linear relations (see [7], [14]).

E-mail address: toddg@union.edu.

https://doi.org/10.1016/j.jnt.2017.09.028
0022-314X/

Please cite this article in press as: G. Todd, A conjectural characterization for $\mathbb{F}_q(t)$-linear relations between multizeta values, J. Number Theory (2018), https://doi.org/10.1016/j.jnt.2017.09.028

Recent research in the function field case has investigated multizeta values in positive characteristic. (See [4] and [13] for a survey of recent results.) Although the function field case does not have two products, we will define two parameterized families of maps between the space of power sum relations. We will posit that these two families of maps, along with one relation, describes all relations. As in the classical case, we restrict our attention to the $\mathbb{F}_q(t)$-span of multizeta values of fixed-weight since it is not expected that there are relations between multizeta values of differing weights [11].

We also provide the following conjecture for the dimension of the span of fixed-weight multizeta values in positive characteristic. Let $g_k = \dim\left(\mathrm{Span}_{\mathbb{F}_q(t)}(\text{MZV's of weight } k)\right)$, then

$$g_k = \begin{cases} 2^{k-1} & \text{if } 1 \le k < q \\ 2^{k-1} - 1 & \text{if } k = q \\ \sum_{i=1}^{q} g_{k-i} & \text{if } k > q. \end{cases}$$

Compare this with Zagier's conjecture for the classical multizeta values: $d_k = d_{k-2} + d_{k-3}$ (see [14]).

We also provide a theorem which shows that the two conjectures in the function field setting are equivalent for all q and weights k with $1 \le k \le \max(q+3, 11)$.

2. Notation

We fix some notation.

$$\begin{aligned} q &= \text{a power of a prime } p \\ A &= \mathbb{F}_q[t] \\ A_+ &= \text{monics in } A \\ A_{\le d} &= \text{elements of } A \text{ of degree at most } d \\ A_{d+} &= \text{monics in } A \text{ of degree exactly } d \\ K &= \mathbb{F}_q(t) \\ K_\infty &= \mathbb{F}_q((1/t)) = \text{completion of } K \text{ at } \infty \\ [n] &= t^{q^n} - t \\ l_n &= \prod_{i=1}^{n} (t - t^{q^i}) \\ L_n &= (-1)^n l_n \\ D_n &= \prod_{i=0}^{n-1} (t^{q^n} - t^{q^i}) \end{aligned}$$

Please cite this article in press as: G. Todd, A conjectural characterization for $\mathbb{F}_q(t)$-linear relations between multizeta values, J. Number Theory (2018), https://doi.org/10.1016/j.jnt.2017.09.028