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The unramified computation of Rankin–Selberg integrals expressed in terms of Bessel models for split orthogonal groups: Part II [☆]

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A R T I C L E I N F O

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This paper is the second part of the paper [S1]. We present the unramified computation of the local integrals, with normalized unramified data, over a p-adic field F, arising from the general Rankin–Selberg integrals for $\mathrm{SO}_m \times \mathrm{GL}_N$, in the Cases 1b, 2 of [S1]. As in the first part, our proof is by "analytic continuation from the unramified computation in the generic case".

1. Introduction

In [S1], we considered three general families of Rankin–Selberg integrals for $\mathrm{SO}_m \times \mathrm{GL}_N$, where SO_m is a split special orthogonal group in m variables. These are the families surveyed in Cases 1a, 1b, 2 in [S1]. We recall them briefly. See [S1] for exact details.

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In all three cases, the global integrals are formed by either integrating a cusp form on an orthogonal group against a Fourier coefficient (of Bessel type) of an Eisenstein series on a bigger orthogonal group, or by integrating a Fourier coefficient of similar type of a cusp form against an Eisenstein series on a smaller orthogonal group. Let π be an irreducible, automorphic, cuspidal representation of $\mathrm{SO}_m(\mathbb{A})$, where $\mathbb{A}$ is the ring of Adeles of a number field F. Assume that π has a (global) Bessel model with respect to an irreducible, automorphic, cuspidal representation σ of $\mathrm{SO}_{m-2k-1}(\mathbb{A})$. In Case 1a, we let τ be an irreducible, automorphic, cuspidal representation of $\mathrm{GL}_{r+k+1}(\mathbb{A})$. The global integrals pair cusp forms from π against Bessel–Fourier coefficients of Eisenstein series on $\mathrm{SO}_{m+2r+1}(\mathbb{A})$, parabolically induced from $\tau|\det\cdot|^{s-\frac{1}{2}}\times\sigma_r$ (σ_r is an outer twist of σ depending on the parity of r). In Case 1b, τ is an irreducible, automorphic, cuspidal representation of $\mathrm{GL}_r(\mathbb{A})$, and the global integrals pair cusp forms from σ against Bessel–Fourier coefficients of Eisenstein series on $\mathrm{SO}_{m+2r}(\mathbb{A})$, parabolically induced from $\tau|\det\cdot|^{s-\frac{1}{2}}\times\pi_r$. In Case 2, the Eisenstein series is on a smaller orthogonal group. Here τ is an irreducible, automorphic, cuspidal representation of $\mathrm{GL}_r(\mathbb{A})$, with $r\le k$, and the global integrals pair Bessel–Fourier coefficients of cusp forms from π against Eisenstein series on $\mathrm{SO}_{m+2(r-k)-1}(\mathbb{A})$, parabolically induced from $\tau|\det\cdot|^{s-\frac{1}{2}}\times\sigma$. The global integrals in all three cases decompose, for decomposable data, into a product of local integrals, and the subject matter of this paper and [S1] is to present "the unramified computation".

From now on, F denotes a p-adic field, where p is an odd prime number; π denotes an irreducible unramified representation of $G=\mathrm{SO}_m(F)$, such that π has a (local) Bessel model with respect to an irreducible, unramified representation σ of $G'_k=\mathrm{SO}_{m-2k-1}(F)$, where $m-2k-1\ge 0$. (When $m-2k-1=0,1$, this means that π is generic, and "there is no σ".) In [S1] we presented the unramified computation of the local integrals corresponding to Case 1a. These local integrals were introduced by Ginzburg, Piatetski-Shapiro and Rallis in [G.PS.R.], where they established the unramified computation when the Bessel model is "spherical", i.e. $k=0$. Let τ be an irreducible, generic, unramified representation of $\mathrm{GL}_{r+k+1}(F)$. Then we proved that the local integrals of Case 1a, when all data are unramified and normalized, are equal to

$$\frac{L(\pi\times\tau,s)}{L(\sigma\times\tau,s+\frac{1}{2})L(\tau,\rho_m,2s)}, \tag{1.1}$$

where $\rho_m=\wedge^2$, if m is odd, and $\rho_m=sym^2$, if m is even. In this paper we address the two remaining cases, Case 2 and Case 1b, in this order. The representations π and σ are as before. In Case 2, τ is an irreducible, generic, unramified representation of $\mathrm{GL}_r(F)$, $1\le r\le k$. The unramified computation of the local integrals of Case 2 gives (1.1).

The local integrals in Case 1b generalize the local integrals studied by Shahidi [Sh] (arising in the Langlands–Shahidi method). They were considered by Friedberg and Goldberg in [F.G.], where they generalized Shahidi's local coefficients. For π and σ as

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