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Schmidt's subspace theorem for moving hypersurface targets

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ABSTRACT

It was discovered that there is a formal analogy between Nevanlinna theory and Diophantine approximation. Via Vojta's dictionary, the Second Main Theorem in Nevanlinna theory corresponds to Schmidt's Subspace Theorem in Diophantine approximation. Recently, Cherry, Dethloff, and Tan (arXiv:1503.08801v2 [math.CV]) obtained a Second Main Theorem for moving hypersurfaces intersecting projective varieties. In this paper, we shall give the counterpart of their Second Main Theorem in Diophantine approximation.

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1. Introduction

Let k be an algebraic number field of degree ρ. Denote by $M(k)$ the set of places (i.e., equivalent classes of absolute values) of k and write $M_\infty(k)$ for the set of Archimedean places. From $v \in M(k)$, we choose the normalized absolute value $|.|_v$ such that $|.|_v = |.|$ on $\mathbb{Q}$ (the standard absolute value) if v is archimedean, whereas for v non-archimedean $|p|_v = p^{-1}$ if v lies above the rational prime p. Denote by k_v the completion of k with respect to v and by $\rho_v = [k_v : \mathbb{Q}_v]$ the local degree. We put $\|.\|_v = |.|_v^{n_v}$, where $n_v = \rho_v/\rho$. Then norm $||.||_v$ satisfies the following properties:

(i) $||x||_v \geq 0$, with equality if and only if $x = 0$;

(ii) $||xy||_v = ||x||_v||y||_v$ for all $x, y \in k$;

(iii) $||x_1 + \cdots + x_m||_v \leq B_v^{n_v} \cdot \max\{||x_1||_v, \ldots, ||x_m||_v\}$ for all $x_1, \ldots, x_m \in k$, $n \in \mathbb{N}$, where $B_v = 1$ if v is non-archimedean and $B_v = n$ if v is archimedean.

Moreover, for each $x \in k \setminus \{0\}$, we have the following product formula:

$$\prod_{v \in M(k)} \|x\|_v = 1.$$

For $v \in M(k)$, we also extend $\|.\|_v$ to an absolute value on the algebraic closure $\overline{k}_v$.

For $x \in k$, the logarithmic height of x is defined by $h(x) = \sum_{v \in M(k)} \log^+ \|x\|_v$, where $\log^+ \|x\|_v = \log \max\{\|x\|_v, 1\}$.

For $x = [x_0 : \cdots : x_M] \in \mathbb{P}^M(k)$, we set $\|x\|_v = \max_{0 \leq i \leq M} \|x_i\|_v$, and define the logarithmic height of x by

$$h(x) = \sum_{v \in M(k)} \log \|x\|_v. \tag{1.1}$$

Notice that the definition of $h(x)$ does not depend on the representative of $x \in \mathbb{P}^M(k)$ because of the product formula.

For a positive integer d, we set

$$\mathcal{T}_d := \big\{(i_0, \ldots, i_M) \in \mathbb{N}_0^{M+1} : i_0 + \cdots + i_M = d\big\}.$$

Let Q be a homogeneous polynomial of degree d in $k[x_0, \ldots, x_M]$. We write

$$Q = \sum_{I \in \mathcal{T}_d} a_I x^I.$$

Set $\|Q\|_v := \max_I \|a_I\|_v$. The height of Q is defined by

$$h(Q) := \sum_{v \in M(k)} \log \|Q\|_v.$$

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