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The stable property of Newton slopes for general Witt towers

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A B S T R A C T

Any polynomial $f(x) \in \mathbb{Z}_q[x]$ defines a Witt vector $[f] \in W(\mathbb{F}_q[x])$. Consider the Artin–Schreier–Witt tower $y^F - y = [f]$. This is a tower of curves over $\mathbb{F}_q$, with total Galois group $\mathbb{Z}_p$. We want to study the Newton slopes of zeta functions of this tower. We reduce it to the Newton polygons of L-functions associated with characters on the Galois groups. We prove that, when the conductors are large enough, these Newton slopes are unions of arithmetic progressions which are changing proportionally as the conductor increases. This is a generalization of the result of [1], where they get the same result in the case the non-zero coefficients of $f(x)$ are roots of unity. To overcome the new difficulty in our process, we apply some (p^θ, T)-topology.

1. Introduction

Let $\mathbb{F}_q$ be a finite field of cardinality $q = p^a$ with p prime. Let $\cdots \to C_n \to C_{n-1} \to \cdots \to C_0 = \mathbb{P}^1_{\mathbb{F}_q}$ be a $\mathbb{Z}_p$-cover of smooth projective geometrically irreducible curves over $\mathbb{F}_q$. This means, for every n, $\mathrm{Gal}(C_n/C_0) \cong \mathbb{Z}/p^n\mathbb{Z}$. For the whole tower $C_\infty := \varprojlim C_n$,

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we have $\mathrm{Gal}(C_\infty/C_0) \cong \mathbb{Z}_p$. Assume that each C_n is totally ramified at ∞ and unramified outside ∞. By the work of M. Kosters and D. Wan [2], this tower corresponds to a power series $f(x) = c_0 + \sum_{i>0,(i,p)=1} c_i x^i \in \mathbb{Z}_q[[x]]$ which is convergent in the unit disk (i.e., $c_i \to 0$ as $i \to \infty$), where $c_0 \in \mathbb{Z}_p$. For every n, C_n is the projective closure of the affine curve defined by the first n equations (with respect to coordinates of Witt vectors) of

$$y^F - y = c_0 + \sum_{i>0,(i,p)=1} c_i [x]^i,$$

where $c_i \in \mathbb{Z}_q \cong W(\mathbb{F}_q)$, $[x] = (x, 0, 0, \ldots), y = (y_1, y_2, \ldots)$ are p-typical Witt vectors, and $\cdot^F$ is the Frobenius map raising every coordinate of a Witt vector to its p-th power. The additions and multiplications in both sides are those of Witt vectors. Without loss of generality, one can assume $c_0 = 0$.

Consider the zeta function of C_n. By Weil's theorems [7],

$$\zeta(C_n, s) = \exp\left(\sum_{k\geq 1} \#C_n(\mathbb{F}_{q^k}) \cdot \frac{s^k}{k}\right) = \frac{P(C_n, s)}{(1-s)(1-qs)}.$$

$P(C_n, s)$ is a polynomial of degree $2g_n$, where g_n is the genus of C_n. In the spirit of Iwasawa theory, it is natural to study the q-adic Newton polygons of the sequence $P(C_n, s)$, especially the behavior of them when $n \to \infty$.

$P(C_n, s)$ is a product of L-functions for various finite characters. Let $\chi : \mathbb{Z}_p \to \mathbb{C}_p^\times$ be a finite additive character. That means the image of χ has finite cardinality. We call this cardinality the conductor of χ. Let m_χ be the nonnegative integer such that the conductor of χ is p^{m_χ}. If χ is nontrivial, we let $\pi_\chi = \chi(1) - 1$, then $\nu_p(\pi_\chi) = \varphi(p^{m_\chi})^{-1}$, where φ is the Euler function. Note that $\mathrm{Gal}(C_{m_\chi}/C_0) \cong \mathbb{Z}/p^{m_\chi}\mathbb{Z}$, so χ can be seen as a homomorphism $\mathrm{Gal}(C_{m_\chi}/C_0) \to \mathbb{C}_p^\times$. Now, we define the L-function $L(\chi, s)$ over $\mathbb{A}^1_{\mathbb{F}_q}$ by

$$L(\chi, s) = \prod_{x \in |\mathbb{A}^1_{\mathbb{F}_q}|} \frac{1}{1 - \chi(\mathrm{Frob}_x) s^{\deg(x)}},$$

where $|\mathbb{A}^1_{\mathbb{F}_q}|$ is the set of closed points of $\mathbb{A}^1_{\mathbb{F}_q}$, and $\mathrm{Frob}_x \in \mathrm{Gal}(C_{m_\chi}/C_0)$ is the Frobenius element associated with x. By Weil's result [7], $L(\chi, s)$ is also a polynomial. In fact, $L(\chi, s) \in 1 + s\mathbb{Z}_p[\pi_\chi][s]$.

For different choices of χ with fixed m_χ, the polynomials $L(\chi, s)$ are conjugated with each other. Denote the degree of $L(\chi, s)$ by d_{m_χ}. After decomposing in $\mathbb{C}_p[s]$ and adjusting the order of factors, we have $L(\chi, s) = \prod_{i=1}^{d_{m_\chi}} (1 - \alpha_i^{(\chi)} s)$ such that $0 < \nu_q(\alpha_1^{(\chi)}) \leq \ldots \leq \nu_q(\alpha_{d_{m_\chi}}^{(\chi)}) < 1$. For fixed m_χ, the q-slope sequence $\{\gamma_i^{(m_\chi)} := \nu_q(\alpha_i^{(\chi)})\}_{1\leq i\leq d_{m_\chi}}$ depends only on m_χ, not on χ.

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