Journal of Number Theory 185 (2018) 434–448

Contents lists available at ScienceDirect

Journal of Number Theory

www.elsevier.com/locate/jnt

Representations by sextenary quadratic forms with coefficients 1, 2, 3 and 6 and on newforms in $S_3(\Gamma_0(24), \chi)$

Zafer Selcuk Aygin

Division of Mathematical Sciences, School of Physical and Mathematical Sciences, Nanyang Technological University, 21 Nanyang Link, Singapore 637371, Singapore

A R T I C L E   I N F O

Article history:
Received 9 May 2017
Received in revised form 16 September 2017
Accepted 18 September 2017
Available online 20 October 2017
Communicated by A. El-Guindy

MSC:
11F11
11F20
11F27
11E20
11E25
11F30

Keywords:
Dedekind eta function
eta quotients
eta products
theta functions
Eisenstein series
Eisenstein forms
Modular forms
Cusp forms
Fourier coefficients
Fourier series

A B S T R A C T

We use the theory of modular forms to give formulas for the number of representations of n by all sextenary quadratic forms with coefficients $1, 2, 3, 6$. We also apply our results to write newforms in $S_3(\Gamma_0(24), \chi)$ in terms of eta quotients.
© 2017 Elsevier Inc. All rights reserved.

E-mail address: selcukaygin@ntu.edu.sg.

https://doi.org/10.1016/j.jnt.2017.09.012
0022-314X/© 2017 Elsevier Inc. All rights reserved.

1. Introduction

Let $\mathbb{N}$, $\mathbb{N}_0$, $\mathbb{Z}$, $\mathbb{C}$ and $\mathbb{H}$ denote the sets of positive integers, non-negative integers, integers, complex numbers and the upper half plane, respectively. We use the notation $q = e(z) := e^{2\pi i z}$ with $z \in \mathbb{H}$, and so $|q| < 1$. Let $k, N \in \mathbb{N}$ and $\Gamma_0(N)$ be the modular subgroup defined by

$$\Gamma_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z},\ ad - bc = 1,\ c \equiv 0 \pmod{N} \right\}.$$

We write $M_k(\Gamma_0(N), \chi)$ to denote the space of modular forms of weight k for $\Gamma_0(N)$ with multiplier χ, and $E_k(\Gamma_0(N), \chi)$ and $S_k(\Gamma_0(N), \chi)$ to denote the subspaces of Eisenstein forms and cusp forms of $M_k(\Gamma_0(N), \chi)$, respectively. It is known (see [25, p. 83], [22, Theorem 2.1.7]) that

$$M_k(\Gamma_0(N), \chi) = E_k(\Gamma_0(N), \chi) \oplus S_k(\Gamma_0(N), \chi). \tag{1.1}$$

Let χ and ψ be primitive characters. For $n \in \mathbb{N}$ we define $\sigma_{(k,\chi,\psi)}(n)$ by

$$\sigma_{(k,\chi,\psi)}(n) = \sum_{1 \le d \mid n} \chi(d)\psi(n/d)d^k. \tag{1.2}$$

If $n \notin \mathbb{N}$ we set $\sigma_{(k,\chi,\psi)}(n) = 0$. For each quadratic discriminant t, we put $\chi_t(n) = \left(\frac{t}{n}\right)$, where $\left(\frac{t}{n}\right)$ is the Kronecker symbol defined by [23, p. 296].

Let χ and ψ be primitive Dirichlet characters such that $\chi(-1)\psi(-1) = -1$ and with conductors $L, R \in \mathbb{N}$, respectively. The weight 3 Eisenstein series are defined by

$$E_{3,\chi,\psi}(z) = c_0 + \sum_{n \ge 1} \sigma_{(2,\chi,\psi)}(n)q^n \tag{1.3}$$

where

$$c_0 = \begin{cases} -B_{3,\chi}/6 & \text{if } R = 1, \\ 0 & \text{if } R > 1, \end{cases}$$

and the generalized Bernoulli numbers $B_{3,\chi}$ attached to χ are defined by the following equation:

$$B_{3,\chi} = 6[x^3] \sum_{a=1}^{L} \frac{\chi(a)xe^{ax}}{e^{Lx} - 1}.$$

From this we compute

$$B_{3,\chi_{-3}} = 2/3,\ B_{3,\chi_{-4}} = 3/2,\ B_{3,\chi_{-8}} = 9,\ B_{3,\chi_{-24}} = 138.$$

Download English Version:

https://daneshyari.com/en/article/8897062

Download Persian Version:

https://daneshyari.com/article/8897062

Daneshyari.com