

# Accepted Manuscript

Small prime solutions of a nonlinear equation

Zhixin Liu

PII: S0022-314X(17)30355-4  
DOI: <https://doi.org/10.1016/j.jnt.2017.09.014>  
Reference: YJNTH 5884

To appear in: *Journal of Number Theory*

Received date: 18 April 2017  
Accepted date: 3 September 2017

Please cite this article in press as: Z. Liu, Small prime solutions of a nonlinear equation, *J. Number Theory* (2018), <https://doi.org/10.1016/j.jnt.2017.09.014>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



## SMALL PRIME SOLUTIONS OF A NONLINEAR EQUATION

ZHIXIN LIU

ABSTRACT. Let  $a_1, \dots, a_4$  be non-zero integers and  $n$  any integer. Suppose that  $a_1, \dots, a_4$  and  $n$  satisfy some related conditions. In this paper we prove that

- (i) if  $a_j$  are not all of the same sign, then the equation  $a_1p_1 + a_2p_2^2 + a_3p_3^2 + a_4p_4^2 = n$  has prime solutions satisfying  $\max\{p_1, p_2^2, p_3^2, p_4^2\} \ll |n| + \max\{|a_j|\}^{14+\varepsilon}$ ;  
(ii) if all  $a_j$  are positive and  $n \gg \max\{|a_j|\}^{15+\varepsilon}$ , then the equation  $a_1p_1 + a_2p_2^2 + a_3p_3^2 + a_4p_4^2 = n$  is soluble in primes  $p_j$ .

## 1. INTRODUCTION

Let  $n$  be an integer, and let  $a_1, \dots, a_4$  be non-zero integers. We consider here the nonlinear equation in the form

$$(1.1) \quad a_1p_1 + a_2p_2^2 + a_3p_3^2 + a_4p_4^2 = n,$$

where  $p_j$  are prime variables. We shall assume the condition of the congruent solubility for (1.1), that is

$$(1.2) \quad N(q) \geq 1 \quad \text{for all } q \geq 1,$$

where

$$N(q) := \text{card}\{(n_1, \dots, n_4) : 1 \leq n_j \leq q, (n_j, q) = 1, \\ a_1n_1 + a_2n_2^2 + a_3n_3^2 + a_4n_4^2 \equiv n \pmod{q}\}.$$

We also suppose that

$$(1.3) \quad (a_i, a_j) = 1, \quad 1 \leq i < j \leq 4,$$

and write  $A = \max\{2, |a_1|, \dots, |a_4|\}$ . The main results in this paper are the following two parallel theorems.

**Theorem 1.1.** *Suppose (1.2) and (1.3). If  $a_1, \dots, a_4$  are not all of the same sign, then (1.1) has solutions in primes  $p_j$  satisfying*

$$\max\{p_1, p_2^2, p_3^2, p_4^2\} \ll |n| + A^{14+\varepsilon},$$

where the implied constant depends only on  $\varepsilon$ .

**Theorem 1.2.** *Suppose (1.2) and (1.3). If  $a_1, \dots, a_4$  are all positive, then (1.1) is soluble whenever*

$$(1.4) \quad n \gg A^{15+\varepsilon},$$

where the implied constant depends only on  $\varepsilon$ .

---

2010 *Mathematics Subject Classification.* 11P32, 11P05, 11P55.

*Key words and phrases.* small prime, Waring-Goldbach problem, circle method.

Download English Version:

<https://daneshyari.com/en/article/8897067>

Download Persian Version:

<https://daneshyari.com/article/8897067>

[Daneshyari.com](https://daneshyari.com)