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DIVISIBILITY ON THE SEQUENCE OF PERFECT SQUARES MINUS ONE: THE GAP PRINCIPLE

TSZ HO CHAN, STEPHEN CHOI, AND PETER CHO-HO LAM

Abstract. In this paper, we consider a gap principle when $a^2-1|b^2-1|c^2-1$ with $1<a<b<c$. As a byproduct, we are led to determine the complete set of pairs of positive integers $1 \le u \le v \le x$ such that $u|v^2-1$ and $v|u^2-1$ and the diophantine equation $u^2+v^2-1=muv$. We also generalize our main theorems to the polynomial $f(n)=A(n+B)^2+C$.

1. Introduction and Main Results

In a previous paper [1], the first author studied the sequence of numbers $f(n) = n^2(n^2+1)$ and asked

Question 1. *Suppose $a^2(a^2+1)$ divides $b^2(b^2+1)$ with $a<b$. Must it be true that there is some gap between a and b? More precisely, is it true that $b > a^{1+\lambda}$ for some small $\lambda > 0$?*

He managed to prove a gap principle with some additional requirements, namely

Theorem 1. *Suppose $a^2(a^2+1)$ divides $b^2(b^2+1)$ with $a<b$, $(a^2, b^2+1) = a^2/(a^2,b^2)$ and $(a^2+1, b^2) = b^2/(a^2,b^2)$. Then*

$$\frac{b}{a} \gg \frac{(\log a)^{1/8}}{(\log\log a)^{12}}.$$

One can also ask the same question for any polynomial with integral coefficients. Here we formulate the question more precisely:

Definition 1. *Let n be a positive integer and $f(x)$ be a polynomial with integral coefficients. Consider the set of all positive integers $a_0 < a_1 < a_2 < \ldots < a_n$ such that $f(a_i)$ divides $f(a_{i+1})$ for $0 \le i \le n-1$. We say that $f(x)$ satisfies the **gap principle of order** n if $\lim a_n/a_0 = \infty$ as $a_0 \to \infty$.*

Note that the set of all such $(a_0, a_1, \ldots, a_n)$ is always infinite since

$$f(a+f(a)) \equiv 0 \pmod{f(a)}.$$

If $f(x)$ satisfies the gap principle of some order n, it will also satisfy the gap principle of any larger order. Therefore we can also make the following definition:

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