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Longest Arithmetic Progressions in Reduced Residue Systems

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Abstract

In this article, we completely determine the length of longest arithmetic progressions in the least positive reduced residue system and in all reduced residue systems modulo n for every positive integer n.

Keywords: arithmetic progression, residue class, prime, squarefree number, asymptotic
2010 MSC: 11B25, 11A07, 11N37, 11N56

1. Introduction

For each $n \in \mathbb{N}$, let $A(n) = \{a \in \mathbb{N} \mid 1 \le a \le n \text{ and } (a, n) = 1\}$ be the least positive reduced residue system modulo n and let $f(n)$ be the length of longest arithmetic progressions contained in $A(n)$. Recamán (see Guy's book [4, Chapter B40]) asks if $f(n) \to \infty$ as $n \to \infty$. P. Stumpf [10] have recently given an affirmative answer to this question by establishing the bounds

$$\max\left\{\frac{p-1}{2}, \frac{n}{\gamma(n)}\right\} \le f(n) \le \max\left\{p-1, \frac{n}{\gamma(n)}\right\},$$

where p is the largest prime factor of n and $\gamma(n) = \prod_{p|n} p$. In this article, we extend his ideas further and obtain exact formula for $f(n)$ for all n (see Theorems 3.1, 3.2, and 3.3). We note that there may be two or more arithmetic progressions contained in $A(n)$ having maximal length. In addition, if we consider the reduced residue system different from $A(n)$, the length of longest arithmetic progressions may be more than $f(n)$. For example, we have $A(10) = \{1, 3, 7, 9\}$ and $f(10) = 2$, while $\{-3, -1, 1, 3\}$ is a set of reduced residue system modulo 10 which contains an arithmetic progression of length $4 > f(10)$. In general, let $B(n)$ be a set of reduced residue system modulo n and let $f_B(n)$ be the length of longest arithmetic progressions contained in $B(n)$. So if $B(10) = \{-3, -1, 1, 3\}$,

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