

# Accepted Manuscript

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PII: S0022-314X(17)30286-X  
DOI: <http://dx.doi.org/10.1016/j.jnt.2017.07.017>  
Reference: YJNTH 5826

To appear in: *Journal of Number Theory*

Received date: 18 April 2017  
Revised date: 5 July 2017  
Accepted date: 16 July 2017

Please cite this article in press as: P. Pongsriiam, Longest arithmetic progressions in reduced residue systems, *J. Number Theory* (2017), <http://dx.doi.org/10.1016/j.jnt.2017.07.017>

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# Longest Arithmetic Progressions in Reduced Residue Systems

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## Abstract

In this article, we completely determine the length of longest arithmetic progressions in the least positive reduced residue system and in all reduced residue systems modulo  $n$  for every positive integer  $n$ .

*Keywords:* arithmetic progression, residue class, prime, squarefree number, asymptotic

*2010 MSC:* 11B25, 11A07, 11N37, 11N56

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## 1. Introduction

For each  $n \in \mathbb{N}$ , let  $A(n) = \{a \in \mathbb{N} \mid 1 \leq a \leq n \text{ and } (a, n) = 1\}$  be the least positive reduced residue system modulo  $n$  and let  $f(n)$  be the length of longest arithmetic progressions contained in  $A(n)$ . Recamán (see Guy's book [4, Chapter B40]) asks if  $f(n) \rightarrow \infty$  as  $n \rightarrow \infty$ . P. Stumpf [10] have recently given an affirmative answer to this question by establishing the bounds

$$\max \left\{ \frac{p-1}{2}, \frac{n}{\gamma(n)} \right\} \leq f(n) \leq \max \left\{ p-1, \frac{n}{\gamma(n)} \right\},$$

where  $p$  is the largest prime factor of  $n$  and  $\gamma(n) = \prod_{p|n} p$ . In this article, we extend his ideas further and obtain exact formula for  $f(n)$  for all  $n$  (see Theorems 3.1, 3.2, and 3.3). We note that there may be two or more arithmetic progressions contained in  $A(n)$  having maximal length. In addition, if we consider the reduced residue system different from  $A(n)$ , the length of longest arithmetic progressions may be more than  $f(n)$ . For example, we have  $A(10) = \{1, 3, 7, 9\}$  and  $f(10) = 2$ , while  $\{-3, -1, 1, 3\}$  is a set of reduced residue system modulo 10 which contains an arithmetic progression of length  $4 > f(10)$ . In general, let  $B(n)$  be a set of reduced residue system modulo  $n$  and let  $f_B(n)$  be the length of longest arithmetic progressions contained in  $B(n)$ . So if  $B(10) = \{-3, -1, 1, 3\}$ ,

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