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Some New Congruences for Andrews' Singular Overpartitions

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Abstract:

Recently, Andrews defined combinatorial objects which he called singular overpartitions and proved that these singular overpartitions which depend on two parameters k and i can be enumerated by the function $\overline{C}_{k,i}(n)$, which denotes the number of overpartitions of n in which no part is divisible by k and only parts $\equiv \pm i \pmod{k}$ may be overlined. G. E. Andrews, S. C. Chen, M. Hirschhorn, J. A. Sellars, Olivia X. M. Yao, M. S. Mahadeva Naika, D. S. Gireesh, Zakir Ahmed and N. D. Baruah noted numerous congruences modulo $2, 3, 4, 6, 12, 16, 18, 32$ and 64 for $\overline{C}_{3,1}(n)$. In this paper, we prove congruences modulo 128 for $\overline{C}_{3,1}(n)$, and congruences modulo 2 for $\overline{C}_{12,3}(n)$, $\overline{C}_{44,11}(n)$,$\overline{C}_{75,15}(n)$, and $\overline{C}_{92,23}(n)$. We also prove "Mahadeva Naika and Gireesh's conjecture", for $n \geq 0$, $\overline{C}_{3,1}(12n+11) \equiv 0 \pmod{144}$ is true.

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Keywords: Singular overpartition, Theta function, Congruence, Dissection.

1 INTRODUCTION

A partition of a positive integer n denoted by $p(n)$, is a nonincreasing sequence of positive integers whose sum is n. If ℓ is a positive integer, then a partition is called a ℓ-regular partition denoted by $b_\ell(n)$, if there is no part divisible by ℓ.
The generating function for $b_\ell(n)$, is given by

$$\sum_{n=0}^{\infty} b_\ell(n)q^n = \frac{(q^\ell;q^\ell)_\infty}{(q;q)_\infty} = \frac{f_\ell}{f_1}, \tag{1.1}$$

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