Some Real Quadratic Number Fields with their Hilbert 2-Class Field having Cyclic 2-Class Group

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Some Real Quadratic Number Fields with their Hilbert 2-Class Field having Cyclic 2-Class Group

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Abstract

Let k be a real quadratic number field with 2-class group $C_2(k)$ isomorphic to $Z/2^mZ \times Z/2^nZ$, $m \geq 1$, $n \geq 2$, and let k^1 be the Hilbert 2-class field of k. We give complete criteria for $C_2(k^1)$ to be cyclic when either d_k, the discriminant of k, is divisible by only positive prime discriminants, or when the 2-class number of k^1 is greater than 2, and partial criteria for $C_2(k^1)$ to be elementary cyclic when d_k is divisible by a negative prime discriminant.

Key Words: real quadratic number field; Hilbert 2-class field; discriminant; 4-rank; unramified quadratic extension; narrow and wide class groups; commutator subgroup; cyclic class group **MSC Classification**: 11R29

Introduction

Let k be an algebraic number field, $C_2(k)$ denote the 2-Sylow subgroup of its ideal class group $C(k)$, and k^1 denote the Hilbert 2-class field of k (in the wide sense). Let k^n (for a nonnegative integer n) be defined inductively as $k^0 = k$ and $k^{n+1} = (k^n)^1$. Let $G = \mathrm{Gal}(k^2/k)$ and G' denote the commutator subgroup of G. By class field theory it is well known that $G/G' \approx \mathrm{Gal}(k^1/k) \approx C_2(k)$, and $G' \approx \mathrm{Gal}(k^2/k^1) \approx C_2(k^1)$. We will us the notation $(2^m, 2^n)$ to denote $Z/2^mZ \times Z/2^nZ$, $m \geq 1$, $n \geq 1$.