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GROWTH OF TORSION OF ELLIPTIC CURVES WITH FULL 2-TORSION OVER QUADRATIC CYCLOTOMIC FIELDS

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ABSTRACT. Let $K = \mathbb{Q}(\sqrt{-3})$ or $\mathbb{Q}(\sqrt{-1})$ and let C_n denote the cyclic group of order n. We study how the torsion part of an elliptic curve over K grows in a quadratic extension of K. In the case $E(K)[2] \approx C_2 \oplus C_2$ we determine how a given torsion structure can grow in a quadratic extension and the maximum number of quadratic extensions in which it grows. We also classify the torsion structures which occur as the quadratic twist of a given torsion structure.

1. Introduction

Let K be a number field. If E/K is an elliptic curve, let $E(K)_{tor}$ denote the torsion part of the group $E(K)$. Much research has been devoted to determining the finite abelian groups (up to isomorphism) which occur as $E(K)_{tor}$ for some elliptic curve E/K. The classification of the possible torsion structures when $K = \mathbb{Q}$ was first completed by Mazur. Let C_n denote the cyclic group of order n.

Theorem 1. *(Mazur [14], Kubert [10]) Let E be an elliptic curve over $\mathbb{Q}$. Then the torsion subgroup $E(\mathbb{Q})_{tor}$ of $E(\mathbb{Q})$ is isomorphic to one of the following 15 groups:*

- C_n *for* $1 \leq n \leq 12, n \neq 11$
- $C_2 \oplus C_{2n}$ *for* $1 \leq n \leq 4$

Furthermore, each group above occurs infinitely often (up to isomorphism).

By contrast, when $K = \mathbb{Q}(\sqrt{5})$, there is only one elliptic curve over K (up to isomorphism) with torsion part C_{15} [18]. The work of Mazur was generalized by Kamienny et al. to the case of quadratic number fields.

Theorem 2. *(Kamienny [6], Kenku and Momose [8]) Let K be a quadratic field and E an elliptic curve over K. Then $E(K)_{tor}$ is isomorphic to one of the following 26 groups:*

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