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THE BEHAVIOR OF DIFFERENTIAL FORMS UNDER PURELY INSEPARABLE EXTENSIONS

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ABSTRACT. Let F be a field of characteristic 2. In this paper we give a complete computation of the kernel of the homomorphism $H_2^{m+1}(F) \longrightarrow H_2^{m+1}(L)$ induced by scalar extension, where L/F is a purely inseparable extension (of any degree), $H_2^{m+1}(F)$ is the cokernel of the Artin-Schreier operator $\wp : \Omega_F^m \longrightarrow \Omega_F^m/\mathrm{d}\Omega_F^{m-1}$ given by: $x\frac{\mathrm{d}x_1}{x_1}\wedge\cdots\wedge\frac{\mathrm{d}x_m}{x_m} \mapsto (x^2-x)\frac{\mathrm{d}x_1}{x_1}\wedge\cdots\wedge\frac{\mathrm{d}x_m}{x_m}+\mathrm{d}\Omega_F^{m-1}$, where Ω_F^m is the space of absolute m-differential forms over F and d is the differential operator. Other related results are included.

1. INTRODUCTION

Let F be a field of characteristic 2. For any integer $m \geq 1$, let $\Omega_F^m = \wedge^m\Omega_F^1$ denote the space of absolute m-differential forms ($\Omega_F^0 = F$), where Ω_F^1 is the F-vector space generated by the symbols $\mathrm{d}x$, $x \in F$, subject to the relations: $\mathrm{d}(x+y) = \mathrm{d}x+\mathrm{d}y$ and $\mathrm{d}(xy) = x\mathrm{d}y+y\mathrm{d}x$ for $x, y \in F$. In particular, there is an F^2-linear map $F \longrightarrow \Omega_F^1$, given by: $x \mapsto \mathrm{d}x$. This map extends to the differential operator $\mathrm{d} : \Omega_F^m \longrightarrow \Omega_F^{m+1}$ given by: $\mathrm{d}(x\mathrm{d}x_1\wedge\cdots\wedge\mathrm{d}x_m) = \mathrm{d}x\wedge\mathrm{d}x_1\wedge\cdots\wedge\mathrm{d}x_m$. The Artin-Schreier operator $\wp : \Omega_F^m \longrightarrow \Omega_F^m/\mathrm{d}\Omega_F^{m-1}$ is defined by: $x\frac{\mathrm{d}x_1}{x_1}\wedge\cdots\wedge\frac{\mathrm{d}x_m}{x_m} \mapsto (x^2-x)\frac{\mathrm{d}x_1}{x_1}\wedge\cdots\wedge\frac{\mathrm{d}x_m}{x_m}+\mathrm{d}\Omega_F^{m-1}$, and the cokernel of this operator is denoted by $H_2^{m+1}(F)$.

For any field extension L/F, we have a group homomorphism $H_2^{m+1}(F) \longrightarrow H_2^{m+1}(L)$ induced by the inclusion $F \subset L$. An important problem consists in computing the kernel $H_2^{m+1}(L/F)$ of this homomorphism. The motivation of considering this problem is the computation of the (graded-)Witt kernel for the extension L/F which reduces to the computation of $H_2^{m+1}(L/F)$ by a celebrated result of Kato [11]. Up to now, the kernel $H_2^{m+1}(L/F)$ is known in the following cases:

(a) L is the function field of a projective F-quadric given by a bilinear Pfister form of arbitrary dimension, or a quadratic Pfister form of dimension 2^k such that $m \leq k$ [3], [7, See after Question 8.1].
(b) L is a quadratic extension of F [2], [3].

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