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Nice derivations over principal ideal domains

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Abstract

In this paper we investigate to what extent the results of Z. Wang and D. Daigle on "nice derivations" of the polynomial ring $k[X, Y, Z]$ over a field k of characteristic zero extend to the polynomial ring $R[X, Y, Z]$ over a PID R, containing the field of rational numbers. One of our results shows that the kernel of a nice derivation on $k[X_1, X_2, X_3, X_4]$ of rank at most three is a polynomial ring over k.

Keywords. Polynomial Rings, Locally Nilpotent Derivation, Nice Derivation.
2010 MSC. Primary: 13N15; Secondary: 14R20, 13A50.

1 Introduction

By a ring, we will mean a commutative ring with unity. Let R be a ring and $n(\geqslant 1)$ be an integer. For an R-algebra A, we use the notation $A = R^{[n]}$ to denote that A is isomorphic to a polynomial ring in n variables over R. We denote the group of units of R by R^*.

Let k be a field of characteristic zero, R a k-domain, $B := R^{[n]}$ and m is a positive integer $\leq n$. In this paper, we consider locally nilpotent derivations D on B, which satisfy $D^2(T_i) = 0$ for all $i \in \{1, \ldots, m\} \subseteq \{1, \ldots, n\}$ for some coordinate system $(T_1, T_2, \ldots, T_n)$ of B. For convenience, we shall call such a derivation D as a *quasi-nice derivation*. In the case $m = n$, such a D is called a *nice derivation* (Thus a nice derivation is also a quasi-nice derivation). We investigate the rank of D when $n = 3$ and R is a PID (see Section 2 for the definition of rank of D).

The case when $B = k^{[3]}$ was investigated by Z. Wang in [14]. He showed that *rank* D is less than 3 for the cases $(m, n) = (2, 3), (3, 3)$ and that *rank* $D = 1$ when D is a nice derivation (i.e., for $(m, n) = (3, 3)$). In [6], Daigle proved that the rank of D is less than 3 even in the case $(m, n) = (1, 3)$ ([6, Theorems 5.1 and 5.2]).

Now let R be a Noetherian domain containing $\mathbb{Q}$, say R is regular. It is natural to ask how far we can extend the above results to $R[X, Y, Z](= R^{[3]})$. In particular, we consider the following question for nice derivations.

Question 1. *If D is a nice derivation of $R[X, Y, Z]$, then is rank $D = 1$, or, at least, is rank $D < 3$?*