ARTICLE IN PRESS JPAA:5832

Journal of Pure and Applied Algebra ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Pure and Applied Algebra

www.elsevier.com/locate/jpaa

Cell structures for the Yokonuma–Hecke algebra and the algebra of braids and ties

Jorge Espinoza [1], Steen Ryom-Hansen [*,2]

Instituto de Matemática y Física, Universidad de Talca, Chile

ARTICLE INFO

Article history:
Received 8 November 2017
Received in revised form 2 January 2018
Available online xxxx
Communicated by S. Donkin

MSC:
33D80

ABSTRACT

We construct a faithful tensor representation for the Yokonuma–Hecke algebra $\mathcal{Y}_{r,n}$, and use it to give a concrete isomorphism between $\mathcal{Y}_{r,n}$ and Shoji's modified Ariki–Koike algebra. We give a cellular basis for $\mathcal{Y}_{r,n}$ and show that the Jucys–Murphy elements for $\mathcal{Y}_{r,n}$ are JM-elements in the abstract sense. Finally, we construct a cellular basis for the Aicardi–Juyumaya algebra of braids and ties.

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1. Introduction

In the present paper, we study the representation theory of the Yokonuma–Hecke algebra $\mathcal{Y}_{r,n}$ in type A and of the related Aicardi–Juyumaya algebra $\mathcal{E}_n$ of braids and ties. In the past few years, quite a few papers have been dedicated to the study of both algebras.

The Yokonuma–Hecke algebra $\mathcal{Y}_{r,n}$ was first introduced in the sixties by Yokonuma [38] for general types but the recent activity on $\mathcal{Y}_{r,n}$ was initiated by Juyumaya who in [22] gave a new presentation of $\mathcal{Y}_{r,n}$. It is a deformation of the group algebra of the wreath product $C_r \wr \mathfrak{S}_n$ of the cyclic group C_r and the symmetric group $\mathfrak{S}_n$. On the other hand, it is quite different from the more familiar deformation of $C_r \wr \mathfrak{S}_n$, the Ariki–Koike algebra $\widetilde{\mathcal{H}}_{r,n}$. For example, the usual Iwahori–Hecke algebra $\mathcal{H}_n$ of type A appears canonically as a quotient of $\mathcal{Y}_{r,n}$, whereas it appears canonically as subalgebra of $\widetilde{\mathcal{H}}_{r,n}$.

Much of the impetus to the recent development on $\mathcal{Y}_{r,n}$ comes from knot theory. In the papers [5], [8], [9], [21] and [23] a Markov trace on $\mathcal{Y}_{r,n}$ and its associated knot invariant Θ is studied.

The Aicardi–Juyumaya algebra $\mathcal{E}_n$ of braids and ties, along with its diagram calculus, was introduced in [1] and [20] via a presentation derived from the presentation of $\mathcal{Y}_{r,n}$. The algebra $\mathcal{E}_n$ is also related to knot

* Corresponding author.
E-mail addresses: joespinoza@utalca.cl (J. Espinoza), steen@inst-mat.utalca.cl (S. Ryom-Hansen).

1 Supported in part by Beca Doctorado Nacional 2013-CONICYT 21130109.
2 Supported in part by FONDECYT grants 1121129 and 1171379.

https://doi.org/10.1016/j.jpaa.2018.02.001
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Please cite this article in press as: J. Espinoza, S. Ryom-Hansen, Cell structures for the Yokonuma–Hecke algebra and the algebra of braids and ties, J. Pure Appl. Algebra (2018), https://doi.org/10.1016/j.jpaa.2018.02.001

theory. Indeed, Aicardi and Juyumaya constructed in [2] a Markov trace on $\mathcal{E}_n$, which gave rise to a three parameter knot invariant Δ. There seems to be no simple relation between Θ and Δ.

A main aim of our paper is to show that $\mathcal{Y}_{r,n}$ and $\mathcal{E}_n$ are cellular algebras in the sense of Graham and Lehrer, [14]. On the way we give a concrete isomorphism between $\mathcal{Y}_{r,n}$ and Shoji's modified Ariki–Koike algebra $\mathcal{H}_{r,n}$. This gives a new proof of a result of Lusztig [25] and Jacon–Poulain d'Andecy [18], showing that $\mathcal{Y}_{r,n}$ is in fact a sum of matrix algebra over Iwahori–Hecke algebras of type A.

For the parameter $q = 1$, it was shown in Banjo's work [4] that the algebra $\mathcal{E}_n$ is a special case of P. Martin's ramified partition algebras, see [27]. Moreover, Marin showed in [26] that $\mathcal{E}_n$ for $q = 1$ is isomorphic to a sum of matrix algebras over a certain wreath product algebra, in the spirit of Lusztig's and Jacon–Poulain d'Andecy's Theorem. He raised the question whether this result could be proved for general parameters. As an application of our cellular basis for $\mathcal{E}_n$ we do obtain such a structure Theorem for $\mathcal{E}_n$, thus answering in the positive Marin's question.

Recently it was shown in [8] and [32] that the Yokonuma–Hecke algebra invariant Θ can be described via a formula involving the HOMFLYPT-polynomial and the linking number. In particular, when applied to classical knots, Θ and the HOMFLYPT-polynomial coincide (this was already known for some time). Given our results on $\mathcal{E}_n$ it would be interesting to investigate whether a similar result would hold for Δ.

Roughly our paper can be divided into three parts. The first part, sections 2 and 3, contains the construction of a faithful tensor space module $V^{\otimes n}$ for $\mathcal{Y}_{r,n}$. The construction of $V^{\otimes n}$ is a generalization of the $\mathcal{E}_n$-module structure on $V^{\otimes n}$ that was defined in [33] and it allows us to conclude that $\mathcal{E}_n$ is a subalgebra of $\mathcal{Y}_{r,n}$ for $r \geq n$, and for *any* specialization of the ground ring. The tensor space module $V^{\otimes n}$ is also related to the strange Ariki–Terasoma–Yamada action, [3] and [34], of the Ariki–Koike algebra on $V^{\otimes n}$, and thereby to the action of Shoji's modified Ariki–Koike algebra $\mathcal{H}_{r,n}$ on $V^{\otimes n}$, [36]. A speculating remark concerning this last point was made in [33], but the appearance of Vandermonde determinants in the proof of the faithfulness of the action of $\mathcal{Y}_{r,n}$ in $V^{\otimes n}$ makes the remark much more precise. The defining relations of the modified Ariki–Koike algebra also involve Vandermonde determinants and from this we obtain the proof of the isomorphism $\mathcal{Y}_{r,n} \cong \mathcal{H}_{r,n}$ by viewing both algebras as subalgebras of $\mathrm{End}(V^{\otimes n})$. Via this, we get a new proof of Lusztig's and Jacon–Poulain d'Andecy's isomorphism Theorem for $\mathcal{Y}_{r,n}$, since it is in fact equivalent to a similar isomorphism Theorem for $\mathcal{H}_{r,n}$, obtained independently by Sawada–Shoji and Hu–Stoll.

The second part of our paper, sections 4 and 5, contains the proof that $\mathcal{Y}_{r,n}$ is a cellular algebra in the sense of Graham–Lehrer, via a concrete combinatorial construction of a cellular basis for it, generalizing Murphy's standard basis for the Iwahori–Hecke algebra of type A. The fact that $\mathcal{Y}_{r,n}$ is cellular could also have been deduced from the isomorphism $\mathcal{Y}_{r,n} \cong \mathcal{H}_{r,n}$ and from the fact that $\mathcal{H}_{r,n}$ is cellular, as was shown by Sawada and Shoji in [35]. Still, the usefulness of cellularity depends to a high degree on having a concrete cellular basis in which to perform calculations, rather than knowing the mere existence of such a basis, and our construction should be seen in this light.

Cellularity is a particularly strong language for the study of modular, that is non-semisimple representation theory, which occurs in our situation when the parameter q is specialized to a root of unity. But here our applications go in a different direction and depend on a nice compatibility property of our cellular basis with respect to a natural subalgebra of $\mathcal{Y}_{r,n}$. We get from this that the elements $m_{\mathfrak{s}\mathfrak{s}}$ of the cellular basis for $\mathcal{Y}_{r,n}$, given by one-column standard multitableaux $\mathfrak{s}$, correspond to certain idempotents that appear in Lusztig's presentation of $\mathcal{Y}_{r,n}$ in [24] and [25]. Using the faithfulness of the tensor space module $V^{\otimes n}$ for $\mathcal{Y}_{r,n}$ we get via this Lusztig's idempotent presentation of $\mathcal{Y}_{r,n}$. Thus the second part of the paper depends logically on the first part.

In section 5 we treat the Jucys–Murphy's elements for $\mathcal{Y}_{r,n}$. They were already introduced and studied by Chlouveraki and Poulain d'Andecy in [7], but here we show that they are JM-elements in the abstract sense defined by Mathas, with respect to the cell structure that we found.

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