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# A Study of Quasi-Gorenstein Rings

EHSAN TAVANFAR and MASSOUD TOUSI

ABSTRACT. This paper is devoted to study certain quasi-Gorenstein counterparts to some known properties of Gorenstein rings.

## 1 Introduction

Throughout,  $(R, \mathfrak{m})$ , stands for a  $d$ -dimensional commutative Noetherian local ring. Following [3], we say that  $R$  is a quasi-Gorenstein ring precisely when  $H_{\mathfrak{m}}^d(R) \cong E(R/\mathfrak{m})$  or, equivalently,  $R$  has a canonical module which is a rank one free module. In the geometric vein, a normal projective variety  $X$  is quasi-Gorenstein if the canonical divisor  $K_X$  of  $X$  is Cartier. Plenty of examples of non-Cohen-Macaulay, i.e. non-Gorenstein, quasi-Gorenstein rings exist, including certain canonical covers, Segre products, invariant rings or affine semigroup rings (see, e.g. [24], [7] and [16]). In the present paper, we investigate certain behavior, or properties, of quasi-Gorenstein rings which are known in the Gorenstein case, but are unknown for the more general setting of quasi-Gorenstein rings. In the Section 2, we study the interaction of quasi-Gorensteinness and regular elements, e.g. we prove the following result.

**Theorem 1.1.** (See Theorem 2.6) *If  $R$  is quasi-Gorenstein and  $x$  is a regular element of  $R$  such that  $R/xR$  has a canonical module, then the  $S_2$ -ification of  $R/xR$  is quasi-Gorenstein.*

We also establish the following variant of deformation of quasi-Gorensteinness.

**Theorem 1.2.** (See Theorem 2.9) *Suppose that  $x \in \mathfrak{m} \setminus Z(R)$  and that  $R/x^n R$  is quasi-Gorenstein for infinitely many  $n \in \mathbb{N}$ . Then  $R$  is quasi-Gorenstein.*

By, e.g. [12], a Cohen-Macaulay local ring is Gorenstein if and only if  $H_{\mathfrak{m}}^d(R)$  has finite injective dimension. In [25], the authors prove that if  $H_{\mathfrak{m}}^d(R)$  has finite Gorenstein injective dimension and  $R$  is Cohen-Macaulay, then  $R$  is Gorenstein. On the other hand, in the light of [2, Theorem 3],  $R$  is a quasi-Gorenstein ring if and only if  $\text{Id}_R(H_{\mathfrak{m}}^d(R)) < \infty$ . Accordingly, we investigate the Gorenstein injective version of [2, Theorem 3]. We present the following result whose proof takes Section 3 and our result recovers the Cohen-Macaulay case of [25].

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