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TILTING MODULES UNDER SPECIAL BASE CHANGES

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ABSTRACT. Given a non-unit, non-zero-divisor, central element x of a ring Λ, it is well known that many properties or invariants of Λ determine, and are determined by, those of $\Lambda/x\Lambda$ and Λ_x. In the present paper, we investigate how the property of "being tilting" behaves in this situation. It turns out that any tilting module over Λ gives rise to tilting modules over Λ_x and $\Lambda/x\Lambda$ after localization and passing to quotient respectively. On the other hand, it is proved that under some mild conditions, a module over Λ is tilting if its corresponding localization and quotient are tilting over Λ_x and $\Lambda/x\Lambda$ respectively.

INTRODUCTION

Convention. Throughout this note, "ring" means an *arbitrary ring with a non-zero unit*, unless otherwise stated. If Λ is a ring, by a "Λ-module" we mean a *left* Λ-module and right Λ-modules are considered as left modules over Λ^{op}—the opposite ring of Λ. An element $x \in \Lambda$ is said to be *regular* on a Λ-module M if it is non-zero-divisor over M and $xM \neq M$.

Notation. Given a ring Λ, the flat and the projective dimension of Λ-modules will be denoted by $\mathrm{fd}_\Lambda(-)$ and $\mathrm{pd}_\Lambda(-)$ respectively. The *global dimension* and *weak global dimension* of Λ will also be denoted by $\mathrm{gldim}(\Lambda)$ and $\mathrm{w.\,gldim}(\Lambda)$ respectively. For a central element $x \in \Lambda$, we denote by M_x the localization of a Λ-module M with respect to the multiplicatively closed subset $\{x^n : n \in \mathbb{N}_0\}$ of Λ. If R is the center of Λ, then for all Λ-modules M and N, $\mathrm{Ext}^n_\Lambda(M,N)$ has an R-module structure and so we may form its localization as an R-module with respect to $\{x^n : n \in \mathbb{N}_0\}$, denoted by $\mathrm{Ext}^n_\Lambda(M,N)_x$.

Tilting theory is one of the major branches of representation theory of algebras which originally was introduced in the context of finitely generated modules over artin algebras, mainly through the work of Brenner and Butler [9], Happel and Ringel [20] and Happel [19]. The theory was later generalized to the setting of finitely generated modules over associated rings by Miyashita [27], and extended to the setting of arbitrary modules over associative rings by Angeleri-Hügel et al. [2, 1]. One of the main themes of tilting theory is to compare the properties of a ring and its tilted ring: Once a tilting module T is detected over a ring Λ, one can look at the "tilted ring" $\mathrm{End}_\Lambda(T)^{\mathrm{op}}$ which has in general many representation-theoretic, (co)homological and K-theoretic properties and invariants in common with the original ring Λ (see e.g. [30, Section 9]). This close connection between Λ and its tilted ring is in large part due to some equivalences induced by T between certain subcategories of the module categories or derived categories of the two rings; see e.g. [27], [18], [30], [6], [7] and also Remark (2.7).

In this note, we aim to study tilting modules under special base changes associated with a central element of a ring. Generally speaking, base change theorems deal with ascent or descent of invariants or properties of rings and their modules along a ring homomorphism $\Lambda \longrightarrow \Lambda'$. As far as tilting theory is concerned, a basic question in this regard is that when for a given tilting Λ-module T, the Λ'-module $\Lambda' \otimes_\Lambda T$ is tilting? This problem has already been addressed—for finitely generated tilting modules—by several authors

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