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Derived categories of graded gentle one-cycle algebras

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ABSTRACT

Let A be a graded algebra. It is shown that the derived category of dg modules over A (viewed as a dg algebra with trivial differential) is a triangulated hull of a certain orbit category of the derived category of graded A-modules. This is applied to study derived categories of graded gentle one-cycle algebras.

1. Introduction

Discrete triangulated categories are, roughly speaking, those Krull–Schmidt triangulated categories which do not admit 'continuous' families of isomorphism classes of indecomposable objects (see [36,12] for various notions of discreteness). A special class of such categories called locally finite triangulated categories (*e.g.* those with finitely many isomorphism classes of indecomposable objects) were intensively studied, in particular, their Auslander–Reiten quivers are classified, see [37,1]. For a finite-dimensional algebra (over an algebraically closed field), Vossieck's theorem [36] states that its derived category is discrete if and only if it is derived equivalent to a hereditary algebra of finite representation type (namely, the path algebra of a Dynkin quiver) or it is a gentle one-cycle algebra which does not satisfy the *clock condition* (see Section 7). The Auslander–Reiten quiver was determined in the former case by Happel in [18] and in the latter case by Bobiński–Geiss–Skowroński in [7]. See [8,4,10,11,30] for further study on discrete derived categories.

Recently, certain discrete triangulated categories of geometrical origin have been studied, *e.g.* the triangulated category generated by a d-spherical object [24] and the relative singularity category of the Auslander resolution of the nodal curve singularity [13]. They turn out to be derived categories of dg modules over

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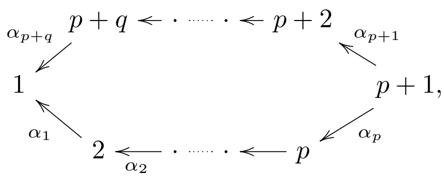
certain graded gentle one-cycle algebras, more precisely, $k[x]/x^2$ with $\deg(x) = d$ and the path algebra of the graded quiver $1 \underset{\beta}{\overset{\alpha}{\rightleftarrows}} 2$ with both arrows in degree -1, respectively. Moreover, derived categories of graded hereditary algebras of type $\tilde{A}_n$ are triangle equivalent to partially wrapped Fukaya categories of graded annuli, see [17, Sections 1.2 and 6.3] and [26, Section 2.1]. Our Theorem 1.2 below gives a representation theoretic description of these triangulated Fukaya categories and gives a partial answer to the following question.

Question. When is the derived category of dg modules over a graded gentle one-cycle algebra (viewed as a dg algebra with trivial differential) discrete and what does the Gabriel/Auslander–Reiten quiver look like?

In this paper we are not able to define derived discreteness for graded algebras, but with Theorem 1.2 we believe that the graded algebras $\Gamma(p, q, r)$ ($r \in \mathbb{Z}\backslash\{0\}$) and $\Gamma'(q, r)$ ($r \in \mathbb{Z}$) in Theorem 1.1 are derived discrete for a reasonable definition of derived discreteness.

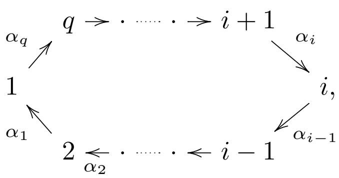
Theorem 1.1. *Let A be a graded gentle one-cycle algebra.*

(a) *If A has finite global dimension, then there is a triple (p, q, r) of integers with $p, q \in \mathbb{N}$[3] and $r \in \mathbb{Z}$ such that A is derived equivalent to the path algebra $\Gamma(p, q, r)$ of the graded quiver*



where $\deg(\alpha_i) = \delta_{i,p+q}r$.

(b) *If A has infinite global dimension, then there are integers $q \in \mathbb{N}$ and $r \in \mathbb{Z}$ such that A is derived equivalent to the quotient algebra $\Gamma'(q, r)$ of the path algebra of the graded quiver*



modulo all paths of length two, where $\deg(\alpha_i) = \delta_{i,q}(q - r)$.

For a dg algebra A, let $\mathcal{D}_{fd}(A)$ denote the full subcategory of the derived category of A consisting of those dg A-modules with finite-dimensional total cohomology.

Theorem 1.2. *Let $p, q \in \mathbb{N}$ and $r \in \mathbb{Z}\backslash\{0\}$.*

(a) *The Auslander–Reiten quiver of $\mathcal{D}_{fd}(\Gamma(p, q, r))$ has $3|r|$ connected components: $\mathcal{X}_i^1$ of type $\mathbb{Z}A_\infty$, $\mathcal{X}_i^2$ of type $\mathbb{Z}A_\infty$ and $\mathcal{P}_i$ of type $\mathbb{Z}A_\infty^\infty$, where $0 \le i \le |r| - 1$. The suspension functor defines cyclic permutations of order $|r|$ on the sets $\{\mathcal{X}_i^1\}$, $\{\mathcal{X}_i^2\}$ and $\{\mathcal{P}_i\}$, respectively. For $X \in \mathcal{X}_i^1$ we have $\tau^p X = \Sigma^r X$ and for*

[3] $\mathbb{N}$ is the set of positive integers.

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