

Accepted Manuscript

Regularity of line configurations

Bruno Benedetti, Michela Di Marca, Matteo Varbaro

PII: S0022-4049(17)30246-3
DOI: <https://doi.org/10.1016/j.jpaa.2017.10.009>
Reference: JPAA 5775

To appear in: *Journal of Pure and Applied Algebra*

Received date: 10 April 2017
Revised date: 1 October 2017

Please cite this article in press as: B. Benedetti et al., Regularity of line configurations, *J. Pure Appl. Algebra* (2017), <https://doi.org/10.1016/j.jpaa.2017.10.009>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



Regularity of line configurations

Bruno Benedetti ¹

Dept. of Mathematics

University of Miami

1365 Memorial Drive

Coral Gables, FL 33146

bruno@math.miami.edu

Michela Di Marca ²

Dip. di Matematica

Università di Genova

Via Dodecaneso 35

16146 Genova, Italy

dimarca@dima.unige.it

Matteo Varbaro ³

Dip. di Matematica

Università di Genova

Via Dodecaneso 35

16146 Genova, Italy

varbaro@dima.unige.it

Dedicated to the memory of Tony Geramita

Abstract

We show that in arithmetically-Gorenstein line arrangements with only planar singularities, each line intersects the same number of other lines. This number has an algebraic interpretation: it is the Castelnuovo–Mumford regularity of the coordinate ring of the arrangement.

We also prove that every $(d - 1)$ -dimensional simplicial complex whose 0-th and 1-st homologies are trivial is the nerve complex of a suitable d -dimensional standard graded algebra of depth ≥ 3 . This provides the converse of a recent result by Katzman, Lyubeznik and Zhang.

Introduction

The study of lines on smooth surfaces of \mathbb{P}^3 has a fascinating history. Lines on smooth cubics were investigated in the Nineteenth century by Cayley [Ca1849], Salmon [Sa1849], and Clebsch [Cl1861], among others. Every smooth cubic contains exactly 27 lines, whose pattern of intersection is independent of the chosen cubic. In 1910 Schoute showed that the 27 lines can be put into a one-to-one correspondence with the vertices of a 6-dimensional polytope, so that all incidence relations between the lines are mirrored in the combinatorial structure of the polytope [Sch1910] [DuV1933].

The situation changes drastically for surfaces of larger degree. In fact, the generic surface of degree $d \geq 4$ contains no line at all. However, special smooth surfaces do contain lines (they are forced to be in a finite number whenever $d \geq 3$ though). An example of a smooth quartic containing 64 lines was found in 1882 by Schur [Sc1882]:

$$x_0^4 - x_0x_1^3 = x_2^4 - x_2x_3^3.$$

¹Supported by NSF Grant 1600741, ‘Geometric Combinatorics and Discrete Morse Theory’

²Supported by PRIN 2010S47ARA 003, ‘Geometria delle Varietà Algebriche.’

³Supported by PRIN 2010S47ARA 003, ‘Geometria delle Varietà Algebriche.’

Download English Version:

<https://daneshyari.com/en/article/8897434>

Download Persian Version:

<https://daneshyari.com/article/8897434>

[Daneshyari.com](https://daneshyari.com)