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Regularity of line configurations

Bruno Benedetti, Michela Di Marca, Matteo Varbaro

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Bruno Benedetti [1]
Dept. of Mathematics
University of Miami
1365 Memorial Drive
Coral Gables, FL 33146
bruno@math.miami.edu

Michela Di Marca [2]
Dip. di Matematica
Università di Genova
Via Dodecaneso 35
16146 Genova, Italy
dimarca@dima.unige.it

Matteo Varbaro [3]
Dip. di Matematica
Università di Genova
Via Dodecaneso 35
16146 Genova, Italy
varbaro@dima.unige.it

Dedicated to the memory of Tony Geramita

Abstract

We show that in arithmetically-Gorenstein line arrangements with only planar singularities, each line intersects the same number of other lines. This number has an algebraic interpretation: it is the Castelnuovo–Mumford regularity of the coordinate ring of the arrangement.

We also prove that every $(d-1)$-dimensional simplicial complex whose 0-th and 1-st homologies are trivial is the nerve complex of a suitable d-dimensional standard graded algebra of depth ≥ 3. This provides the converse of a recent result by Katzman, Lyubeznik and Zhang.

Introduction

The study of lines on smooth surfaces of $\mathbb{P}^3$ has a fascinating history. Lines on smooth cubics were investigated in the Nineteenth century by Cayley [Ca1849], Salmon [Sa1849], and Clebsch [Cl1861], among others. Every smooth cubic contains exactly 27 lines, whose pattern of intersection is independent of the chosen cubic. In 1910 Schoute showed that the 27 lines can be put into a one-to-one correspondence with the vertices of a 6-dimensional polytope, so that all incidence relations between the lines are mirrored in the combinatorial structure of the polytope [Sch1910] [DuV1933].

The situation changes drastically for surfaces of larger degree. In fact, the generic surface of degree $d \geq 4$ contains no line at all. However, special smooth surfaces do contain lines (they are forced to be in a finite number whenever $d \geq 3$ though). An example of a smooth quartic containing 64 lines was found in 1882 by Schur [Sc1882]:

$$x_0^4 - x_0x_1^3 = x_2^4 - x_2x_3^3.$$

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