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Abstract

Using a new equivalent definition of support varieties in the sense of Snashall and Solberg [SS04], we show that both the **(Fg)** condition and support varieties are preserved under singular equivalences of Morita type. In particular, support variety theory is invariant under stable equivalences of Morita type.

1 Introduction

Support is a fundamental concept to provide a geometric approach for studying all kinds of algebraic structures. Based on Quillen's work, Carlson introduced the notion of a support variety of any finitely generated module over a finite group G . The dimension of a support variety is given by the complexity of a module, the rate of growth of the minimal projective resolution of a module [Car83]. Many interesting results are also obtained for p -Lie algebras, finite group schemes and complete intersection rings (see [FP86, FS97, AB00]). In [SS04], Snashall and Solberg developed and generalized the theory of support varieties to finite dimensional algebras, in terms of the maximal ideal spectrum of some commutative graded subring of cohomology operators with a certain finite generation condition, called the **(Fg)** condition. The behavior of a support variety allows to read off homological information on the module itself.

The purpose of this paper is to clarify the behavior of support variety theory under singular equivalences. Let A and B be finite dimensional algebras over a field k . The singularity category $D_{sg}(A)$ is defined to be the Verdier quotient of the bounded derived category of finitely generated left A -modules by the full subcategory of perfect complexes. It is clear that $D_{sg}(A)$ is a triangulated category, and $D_{sg}(A)$ is trivial if and only if A has finite global dimension. A fundamental result of Buchweitz and Happel is that, for a Gorenstein algebra A , the singularity category $D_{sg}(A)$ is triangle equivalent to the stable category of maximal Cohen-Macaulay modules [Buc, Hap91]. This result generalized Rickard's result for self-injective algebras [Ric89a]. The algebras A and B are singularly equivalent if their singularity categories are equivalent as triangulated categories. Derived equivalent algebras are singularly equivalent. However, the converse is wrong. In [KPS], Julian Külshammer, Chrysostomos Psaroudakis and Øystein Skartsaetherhagen proved that the **(Fg)** condition and support varieties are preserved under derived equivalences. This conclusion is false for singular equivalences, because singular equivalences do not preserve Gorensteinness and the **(Fg)** condition (see [PSS14, Example 5.5]). In this paper, we concentrate on a special kind of singular equivalences which are called singular equivalences of Morita type. This notion was proposed by Xiao-Wu Chen and Longgang Sun in [CS]. Singular equivalences of Morita type are induced by bimodules, and capture rich structural information. They generalise

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