



Contents lists available at ScienceDirect

Journal of Pure and Applied Algebra

www.elsevier.com/locate/jpaa

Copointed Hopf algebras over $\mathbb{S}_4$ [☆]Agustín García Iglesias ^{*}, Cristian VayFaMAF-CIEM (CONICET), Universidad Nacional de Córdoba, Medina Allende s/n,
Ciudad Universitaria, 5000 Córdoba, Argentina

ARTICLE INFO

Article history:

Received 20 June 2017

Received in revised form 22

September 2017

Available online xxxx

Communicated by A. Solotar

MSC:

16T05

ABSTRACT

We study the realizations of certain braided vector spaces of rack type as Yetter–Drinfeld modules over a cosemisimple Hopf algebra H . We apply the strategy developed in [1] to compute their liftings and use these results to obtain the classification of finite-dimensional copointed Hopf algebras over $\mathbb{S}_4$.

© 2017 Elsevier B.V. All rights reserved.

1. Introduction

A braided vector space is a pair (V, c) where V is a vector space and $c \in \text{GL}(V \otimes V)$ is a map satisfying the *braid equation*

$$(c \otimes \text{id})(\text{id} \otimes c)(c \otimes \text{id}) = (\text{id} \otimes c)(c \otimes \text{id})(\text{id} \otimes c).$$

If H is a Hopf algebra, a *realization of V over H* is an structure of Yetter–Drinfeld H -module on the vector space V in such a way that the braiding c coincides with the categorical braiding of V as an object in ${}^H_H\mathcal{YD}$.

A *lifting* of $V \in {}^H_H\mathcal{YD}$ is a Hopf algebra A such that $\text{gr } A$ is isomorphic to $\mathfrak{B}(V) \# H$, that is the bosonization of the Nichols algebra of V with H . In particular $H \simeq A_{(0)}$, the coradical of A , and (V, c) is said to be the *infinitesimal braiding* of A .

In [1] we developed a strategy to compute the liftings of a given $V \in {}^H_H\mathcal{YD}$ as cocycle deformations of $\mathfrak{B}(V) \# H$. In a few words, the strategy produces a family of H -module algebras $\mathcal{E}(\lambda)$, obtained as deformations of $\mathfrak{B}(V)$. Set $\mathcal{H} = \mathfrak{B}(V) \# H$. If $\mathcal{E}(\lambda) \neq 0$, then $\mathcal{A}(\lambda) = \mathcal{E}(\lambda) \# H$ is an $\mathcal{H}$ -cleft object and the associated Schauenburg's left Hopf algebra $\mathcal{L}(\lambda) = L(\mathcal{A}(\lambda), \mathcal{H})$ is a lifting of $V \in {}^H_H\mathcal{YD}$.

[☆] The work was partially supported by CONICET, FONCyT PICT 2015-2854 and 2016-3957, Secyt (UNC), the MathAmSud project GR2HOPF.

^{*} Corresponding author.

E-mail addresses: aigarcia@famaf.unc.edu.ar (A. García Iglesias), vay@famaf.unc.edu.ar (C. Vay).

In particular, the algebras $\mathcal{E}(\lambda)$ are deformations of the Nichols algebra itself, and do not depend a priori of the realization, in the sense that they can be defined for generic parameters. The choice of a realization thus brings a restriction on these parameters as a second step.

Hence this new approach reduces the lifting problem to checking that certain algebras are non-zero. This technical step is solved for the cases we study here by means of computer program [16] and the package [17].

In the present article, we follow this strategy to investigate the quadratic deformations of a Nichols algebra $\mathfrak{B}(V)$ where V is a braided vector space of rack type $V(X, q)$ or $W(q, X)$, cf. §2.4.1. We give a necessary condition to realize such a V over a Hopf algebra and we explore how the quadratic relations of $\mathfrak{B}(V)$ are deformed. As a byproduct, we deduce the quadratic relations of $\mathfrak{B}(W(X, q))$, using [15] and the corresponding description of $\mathfrak{B}(V(X, q))$ given in [12].

This general framework allows us to obtain new classification results about pointed and copointed Hopf algebras. We recall that a Hopf algebra A is said to be *pointed* if $A_{(0)} = \mathbb{k}G(A)$ and *copointed* when $A_{(0)} = \mathbb{k}^G$ for some non-abelian group G . Our main result is the following.

Theorem 1.1. *Let L be a finite-dimensional copointed Hopf algebra over $\mathbb{k}^{\mathbb{S}_4}$, $L \not\cong \mathbb{k}^{\mathbb{S}_4}$. Then L is isomorphic to one and only one of the algebras in the following list:*

- (a) $\mathcal{H}_{[c]}$, $c \in \mathfrak{A}$, cf. Definition 6.10.
- (b) $\mathcal{H}_{[c]}^X$, $c \in \mathfrak{A}$, cf. Definition 6.11.
- (c) $\tilde{\mathcal{H}}_{[c]}$, $c \in \tilde{\mathfrak{A}}$, cf. Definition 6.13.

In particular, L is a cocycle deformation of $\text{gr } L$.

Finite-dimensional copointed Hopf algebras over $\mathbb{k}^{\mathbb{S}_3}$ are classified in [8].

Proof. Finite-dimensional Nichols algebras over $\mathbb{S}_4$ are classified in [6, Theorem 4.7] and every such L is generated in degree one by [3, Theorem 2.1]. The algebras listed in the theorem are a complete family of deformations of these Nichols algebras by Propositions 6.12 and 6.14. The liftings are constructed using the strategy in [1], so they arise as cocycle deformations of their graded versions, see Propositions 6.4. $\square$

1.1. Pointed Hopf algebras

We fix the following list of pairs (X, q) of a rack X and a 2-cocycle $q \in Z^2(X, \mathbb{k})$, see §2.6 for unexplained notation:

- (i) The conjugacy class of transpositions $\mathcal{O}_2^4 \subset \mathbb{S}_4$, $q \equiv -1$;
- (ii) The conjugacy class of transpositions $\mathcal{O}_2^4 \subset \mathbb{S}_4$, $q = \chi$;
- (iii) The conjugacy class of 4-cycles $\mathcal{O}_4^4 \subset \mathbb{S}_4$, $q \equiv -1$.

We turn our attention to pointed Hopf algebras and extend some classification results about pointed Hopf algebras over $\mathbb{S}_4$ to any group with a realization of the right braided vector space. Some of these algebras have been considered previously in the literature, although not with this generality. This is the content of the next theorem.

See (28) for the presentation of the Hopf algebras $\mathcal{H}(\lambda)$ associated to each family of parameters $\lambda \in \Lambda(X, q)$.

Download English Version:

<https://daneshyari.com/en/article/8897460>

Download Persian Version:

<https://daneshyari.com/article/8897460>

[Daneshyari.com](https://daneshyari.com)