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Infinite-Dimensional Triangularization

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Abstract

The goal of this paper is to generalize the theory of triangularizing matrices to linear transformations of an arbitrary vector space, without placing any restrictions on the dimension of the space or on the base field. We define a transformation T of a vector space V to be *triangularizable* if V has a well-ordered basis such that T sends each vector in that basis to the subspace spanned by basis vectors no greater than it. We then show that the following conditions (among others) are equivalent: (1) T is triangularizable, (2) every finite-dimensional subspace of V is annihilated by $f(T)$ for some polynomial f that factors into linear terms, (3) there is a maximal well-ordered set of subspaces of V that are invariant under T, (4) T can be put into a crude version of the Jordan canonical form. We also show that any finite collection of commuting triangularizable transformations is simultaneously triangularizable, we describe the closure of the set of triangularizable transformations in the standard topology on the algebra of all transformations of V, and we extend to transformations that satisfy a polynomial the classical fact that the double-centralizer of a matrix is the algebra generated by that matrix.

Keywords: triangular matrix, linear transformation, simultaneous triangularization, canonical form, function topology, endomorphism ring, locally artinian module, double-centralizer
2010 MSC numbers: 15A04, 15A21 (primary), 16S50, 16W80 (secondary)

1 Introduction

The following summarizes much of the existing wisdom on triangularizing a linear transformation of a finite-dimensional vector space. Our main goal is to generalize this to transformations of vector spaces of arbitrary dimension over an arbitrary field.

Theorem 1 (Classical Triangularization Theorem)**.** *Let k be a field, V a finite-dimensional k-vector space, and T a linear transformation of V. Then the following are equivalent.*