An exact upper bound for sums of element orders in non-cyclic finite groups

Marcel Herzog, Patrizia Longobardi, Mercede Maj

PII: S0022-4049(17)30165-2
DOI: http://dx.doi.org/10.1016/j.jpaa.2017.07.015
Reference: JPAA 5720

To appear in: *Journal of Pure and Applied Algebra*

Received date: 6 September 2016
Revised date: 19 May 2017

Please cite this article in press as: M. Herzog et al., An exact upper bound for sums of element orders in non-cyclic finite groups, *J. Pure Appl. Algebra* (2017), http://dx.doi.org/10.1016/j.jpaa.2017.07.015

An exact upper bound for sums of element orders in non-cyclic finite groups

Marcel Herzog[a], Patrizia Longobardi[b,*], Mercede Maj[b]

[a] *Department of Mathematics, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv, Israel*
[b] *Dipartimento di Matematica, Università di Salerno, via Giovanni Paolo II, 132, 84084 Fisciano (Salerno), Italy*

Abstract

Denote the sum of element orders in a finite group G by $\psi(G)$ and let C_n denote the cyclic group of order n. Suppose that G is a non-cyclic finite group of order n and q is the least prime divisor of n. We proved that $\psi(G) \leq \frac{7}{11}\psi(C_n)$ and $\psi(G) < \frac{1}{q-1}\psi(C_n)$. The first result is best possible, since for each $n = 4k$, k odd, there exists a group G of order n satisfying $\psi(G) = \frac{7}{11}\psi(C_n)$ and the second result implies that if G is of odd order, then $\psi(G) < \frac{1}{2}\psi(C_n)$. Our results improve the inequality $\psi(G) < \psi(C_n)$ obtained by H. Amiri, S.M. Jafarian Amiri and I.M. Isaacs in 2009, as well as other results obtained by S.M. Jafarian Amiri and M. Amiri in 2014 and by R. Shen, G. Chen and C. Wu in 2015. Furthermore, we obtained some $\psi(G)$-based sufficient conditions for the solvability of G.

Keywords: Group element orders, Solvable groups
2010 MSC: 20D60, 20E34, 20F16

1. Introduction

The problem of detecting structural properties of a periodic group by looking at element orders has been considered by various authors, from many different points of view. For example, if we denote by $\omega(G)$ the set of the orders of all

*corresponding author
Email addresses: herzogm@post.tau.ac.il (Marcel Herzog), plongobardi@unisa.it (Patrizia Longobardi), mmaj@unisa.it (Mercede Maj)