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Minimal star-varieties of polynomial growth and bounded colength

Daniela La Mattina[1,◊], Thais Silva do Nascimento[2,3] and Ana Cristina Vieira[3,*,♮]

[1] *Dipartimento di Matematica e Informatica, Università di Palermo, Via Archirafi, 34, 90123, Palermo, Italy.*
[2] *Departamento de Matemática, Instituto de Ciências Exatas e da Terra, Universidade Federal de Mato Grosso, Av. Fernando Corrêa da Costa, 2367, Cuiabá, Brazil.*
[3] *Departamento de Matemática, Instituto de Ciências Exatas, Universidade Federal de Minas Gerais, Av. Antônio Carlos, 6627, Belo Horizonte, Brazil.*

Abstract. Let $\mathcal{V}$ be a variety of associative algebras with involution $*$ over a field F of characteristic zero. Giambruno and Mishchenko proved in [**6**] that the $*$-codimension sequence of $\mathcal{V}$ is polynomially bounded if and only if $\mathcal{V}$ does not contain the commutative algebra $D = F \oplus F$, endowed with the exchange involution, and M, a suitable 4-dimensional subalgebra of the algebra of 4×4 upper triangular matrices, endowed with the reflection involution. As a consequence the algebras D and M generate the only varieties of almost polynomial growth. In [**20**] the authors completely classify all subvarieties and all minimal subvarieties of the varieties $\mathrm{var}^*(D)$ and $\mathrm{var}^*(M)$. In this paper we exhibit the decompositions of the $*$-cocharacters of all minimal subvarieties of $\mathrm{var}^*(D)$ and $\mathrm{var}^*(M)$ and compute their $*$-colengths. Finally we relate the polynomial growth of a variety to the $*$-colengths and classify the varieties such that their sequence of $*$-colengths is bounded by three.

1. Introduction

Let A be an associative algebra with involution ($*$-algebra) over a field F of characteristic zero and let $c_n^*(A), n = 1, 2, \ldots,$ be its sequence of $*$-codimensions. In case A satisfies a nontrivial identity, it was proved in [**8**] that $c_n^*(A)$ is exponentially bounded. In order to capture the exponential rate of growth of the sequence of $*$-codimensions, recently, in [**7**] the authors proved that for any associative $*$-algebra A, satisfying an ordinary identity,

$$\exp^*(A) = \lim_{n\to\infty} \sqrt[n]{c_n^*(A)}$$

exists and is an integer called the $*$-exponent of A.

Given a variety of $*$-algebras $\mathcal{V}$, the growth of $\mathcal{V}$ is the growth of the sequence of $*$-codimensions of any algebra A generating $\mathcal{V}$, i.e., $\mathcal{V} = \mathrm{var}^*(A)$. In this paper we are interested in varieties of polynomial growth, i.e., varieties of $*$-algebras such that $c_n^*(\mathcal{V}) = c_n^*(A)$ is polynomially bounded.

In such a case, if A is an algebra with 1, in [**19**] it was proved that $c_n^*(A) = qn^k + O(n^{k-1})$ is a polynomial with rational coefficients whose leading term satisfies the inequalities $\frac{1}{k!} \le q \le \sum_{i=0}^{k} 2^{k-i}\frac{(-1)^i}{i!}$.

In case of polynomial growth Giambruno and Mishchenko proved in [**6**] that a variety $\mathcal{V}$ has polynomial growth if and only if $\mathcal{V}$ does not contain the commutative algebra $D = F \oplus F$, endowed with the exchange involution, and M, a suitable 4-dimensional subalgebra of the algebra of 4×4 upper triangular matrices, endowed with the reflection involution. As a consequence the $*$-algebras D and M generate the only varieties of almost polynomial growth, i.e, they grow exponentially but any proper subvariety is polynomially bounded.

In [**20**] the authors completely classify all subvarieties of the varieties $\mathrm{var}^*(D)$ and $\mathrm{var}^*(M)$. They also classify all their minimal subvarieties of polynomial growth. We recall that $\mathcal{V}$ is a minimal variety of polynomial growth n^k if asymptotically $c_n^*(\mathcal{V}) \approx an^k$, for some $a \neq 0$, and $c_n^*(\mathcal{U}) \approx bn^t$, with $t < k$, for any proper subvariety $\mathcal{U}$ of $\mathcal{V}$.

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*Corresponding author.
E-mail addresses: daniela.lamattina@unipa.it (La Mattina), thais2909@ufmg.br (Nascimento), anacris@ufmg.br (Vieira).