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Equations defining the multi-Rees algebras of powers of an ideal

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Abstract

In this paper we describe the defining equations of the multi-Rees algebra $R[\{I^{n_i}t_i\}_{1\leq i\leq r}]$ over a Noetherian ring R when I is an ideal of linear type. This generalizes a result of Ribbe and recent work of Lin-Polini and Sosa.

Keywords: Rees Algebra, Veronese Subring, Linear Type
2000 MSC: 13A30

1. Introduction

Let R be a Noetherian ring and $I \subseteq R$ be an ideal of R. An important problem in the theory of Rees rings is to describe $R[It]$ in terms of generators and relations: find an ideal $\mathscr{L}$ in a polynomial ring $S = R[T_1, \ldots, T_n]$ such that $R[It] \cong S/\mathscr{L}$. Generators of the ideal $\mathscr{L}$ are called equations of the Rees algebra. More generally, one would like to describe equations of the multi-Rees algebra $R[I_1t_1, I_2t_2, \ldots, I_rt_r]$. Ribbe [1] describes equations of the multi-Rees algebra $R[It_1, It_2, \ldots, It_r]$ when I is of linear type and he also determines the relation type of the multi-Rees algebra $R[I^{n_1}t_1, \ldots, I^{n_r}t_r]$ when $n_i \geq 2$. Also, Johnson and McLoud-Mann [2] determine the defining equations of the Rees algebra $R[I^nt]$ when I is of linear type. In the work of Lin and Polini [3] these equations are described for $R[I^{n_1}t_1, \ldots, I^{n_r}t_r]$, when $R = k[x_1, \ldots, x_n]$ (k a field) and $I = \langle x_1, \ldots, x_n\rangle$. Sosa [4] describes the equations of the multi-Rees algebras $R[I_1t_1, I_2t_2, \ldots, I_rt_r]$ when R is a polynomial ring over a field and I_i are monomial ideals with some special properties.

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