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Abstract

This paper continues the investigation of isotropy theory for toposes. We develop the theory of isotropy quotients of toposes, culminating in a structure theorem for a class of toposes we call *locally anisotropic*. The theory has a natural interpretation for inverse semigroups, which clarifies some aspects of how inverse semigroups and toposes are related.

Keywords: Topos theory, Inverse semigroups
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1. Introduction

Isotropy theory for toposes [5] has its origins in the theory of inverse semigroups and étale groupoids, emerging from an explanation of how the idempotent centralizer (§ 5.3) of an inverse semigroup is a Morita invariant [6]. Indeed, every Grothendieck topos has internal to it a canonical group object Z called its *isotropy group* [5]. This group classifies isotropy in the sense that for any object X of a topos $\mathscr{E}$, morphisms $X \to Z$ of $\mathscr{E}$ are in natural bijection with natural automorphisms of the so-called étale geometric morphism

$$\mathscr{E}/X \longrightarrow \mathscr{E} \tag{1}$$

associated with X. Such an isotropy automorphism is given by a natural automorphism of its inverse image functor

$$X^* : \mathscr{E} \longrightarrow \mathscr{E}/X\,,\ X^*(E) = E \times X \to X\,,$$

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