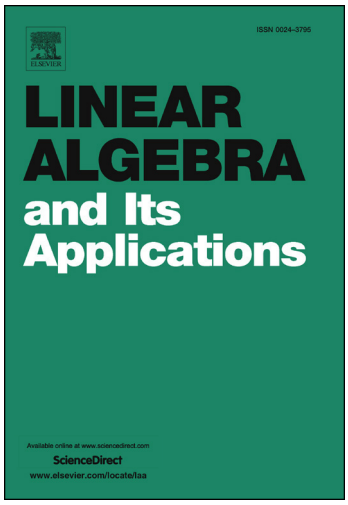

Spectral characterizations of anti-regular graphs

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Spectral characterizations of anti-regular graphs

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Abstract

We study the eigenvalues of the unique connected anti-regular graph A_n. Using Chebyshev polynomials of the second kind, we obtain a trigonometric equation whose roots are the eigenvalues and perform elementary analysis to obtain an almost complete characterization of the eigenvalues. In particular, we show that the interval $\Omega = [\frac{-1-\sqrt{2}}{2}, \frac{-1+\sqrt{2}}{2}]$ contains only the trivial eigenvalues $\lambda = -1$ or $\lambda = 0$, and any closed interval strictly larger than Ω will contain eigenvalues of A_n for all n sufficiently large. We also obtain bounds for the maximum and minimum eigenvalues, and for all other eigenvalues we obtain interval bounds that improve as n increases. Moreover, our approach reveals a more complete picture of the bipartite character of the eigenvalues of A_n, namely, as n increases the eigenvalues are (approximately) symmetric about the number $-\frac{1}{2}$. We also obtain an asymptotic distribution of the eigenvalues as $n \to \infty$. Finally, the relationship between the eigenvalues of A_n and the eigenvalues of a general threshold graph is discussed.

Keywords: adjacency matrix; threshold graph; antiregular graph; Chebyshev polynomials; Toeplitz matrix

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