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Robust group lasso: Model and recoverability ☆

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This paper considers the problem of recovering a group sparse signal matrix $\mathbf{Y} = [\mathbf{y}_1, \cdots, \mathbf{y}_L]$ from sparsely corrupted measurements $\mathbf{M} = [\mathbf{A}_{(1)}\mathbf{y}_1, \cdots, \mathbf{A}_{(L)}\mathbf{y}_L] + \mathbf{S}$, where $\mathbf{A}_{(i)}$'s are known sensing matrices and $\mathbf{S}$ is an unknown sparse error matrix. A robust group lasso (RGL) model is proposed to recover $\mathbf{Y}$ and $\mathbf{S}$ through simultaneously minimizing the $\ell_{2,1}$-norm of $\mathbf{Y}$ and the ℓ_1-norm of $\mathbf{S}$ under the measurement constraints. We prove that $\mathbf{Y}$ and $\mathbf{S}$ can be exactly recovered from the RGL model with high probability for a very general class of $\mathbf{A}_{(i)}$'s.

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1. Introduction

Consider the problem of recovering a group sparse matrix $\mathbf{Y} = [\mathbf{y}_1, \cdots, \mathbf{y}_L] \in \mathbb{R}^{n\times L}$ from sparsely corrupted measurements

$$\mathbf{M} = [\mathbf{A}_{(1)}\mathbf{y}_1, \cdots, \mathbf{A}_{(L)}\mathbf{y}_L] + \mathbf{S}, \tag{1}$$

where $\mathbf{M} = [\mathbf{m}_1, \cdots, \mathbf{m}_L] \in \mathbb{R}^{m\times L}$ is a measurement matrix, $\mathbf{A}_{(i)} \in \mathbb{R}^{m\times n}$ is the i-th sensing matrix, and $\mathbf{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_L] \in \mathbb{R}^{m\times L}$ is an unknown sparse error matrix. The error matrix $\mathbf{S}$ is sparse as it has only a small number of nonzero entries. The signal matrix $\mathbf{Y}$ is group sparse, meaning that $\mathbf{Y}$ is sparse and its nonzero entries appear in a small number of common rows.

Given $\mathbf{M}$ and $\mathbf{A}_{(i)}$'s, our goal is to recover $\mathbf{Y}$ and $\mathbf{S}$ from the linear measurement equation (1). In this paper, we propose to accomplish the recovery task through solving the following robust group lasso (RGL) model

$$\min_{\mathbf{Y},\mathbf{S}} \quad \|\mathbf{Y}\|_{2,1} + \lambda\|\mathbf{S}\|_1, \tag{2}$$

$$s.t. \quad \mathbf{M} = [\mathbf{A}_{(1)}\mathbf{y}_1, \cdots, \mathbf{A}_{(L)}\mathbf{y}_L] + \mathbf{S}. \tag{3}$$

Denoting y_{ij} and s_{ij} as the (i, j)-th entries of $\mathbf{Y}$ and $\mathbf{S}$, respectively, $\|\mathbf{Y}\|_{2,1} \triangleq \sum_{i=1}^{n} \sqrt{\sum_{j=1}^{L} y_{ij}^2}$ is defined as the $\ell_{2,1}$-norm of $\mathbf{Y}$ and $\|\mathbf{S}\|_1 \triangleq \sum_{i=1}^{m}\sum_{j=1}^{L} |s_{ij}|$ is defined as the ℓ_1-norm of $\mathbf{S}$. Minimizing the $\ell_{2,1}$-norm term promotes group sparsity of $\mathbf{Y}$ while minimizing the ℓ_1-norm term promotes sparsity of $\mathbf{S}$; λ is a nonnegative parameter to balance the two terms. We prove that solving the RGL model (2)–(3), which is a convex program, enables exact recovery of $\mathbf{Y}$ and $\mathbf{S}$ with high probability, given that $\mathbf{A}_{(i)}$'s satisfy certain conditions.

1.1. From group lasso to robust group lasso

Sparse signal recovery has attracted much research interest in the signal processing and optimization communities during the past few years. Various sparsity models have been proposed to better exploit the sparse structures of high-dimensional data, such as sparsity of a vector [1], [2], group sparsity of vectors [3], and low-rankness of a matrix [4]. For more topics related to sparse signal recovery, readers are referred to the recent survey paper [5].

In this paper we are interested in the recovery of group sparse (also known as block sparse [6] or jointly sparse [7]) signals which finds a variety of applications such as direction-of-arrival estimation [8], [9], collaborative spectrum sensing [10–12] and motion detection [13]. A signal matrix $\mathbf{Y} = [\mathbf{y}_1, \cdots, \mathbf{y}_L] \in \mathbb{R}^{n\times L}$ is called k-group sparse if k rows of $\mathbf{Y}$ are nonzero. A measurement matrix $\mathbf{M} = [\mathbf{m}_1, \cdots, \mathbf{m}_L] \in \mathbb{R}^{m\times L}$ is taken from linear projections $\mathbf{m}_i = \mathbf{A}_{(i)}\mathbf{y}_i$, $i = 1, \cdots, L$, where $\mathbf{A}_{(i)} \in \mathbb{R}^{m\times n}$ is a sensing matrix. In order to recover $\mathbf{Y}$ from $\mathbf{A}_{(i)}$'s and $\mathbf{M}$, the standard $\ell_{2,1}$-norm minimization formulation proposes to solve a convex program