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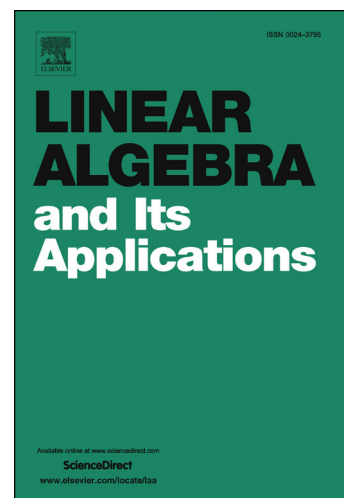
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Restrictions on the Schmidt rank of bipartite unitary operators beyond dimension two

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Abstract

There are none.

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1. Introduction

In the theory of quantum information [2], several notions of rank have been studied. The most prominent such notion is the Schmidt rank of pure bipartite quantum states, which plays an important role in entanglement theory [13, Section VI-A]. In this work, we study the possible values of a similar quantity for bipartite operators, the *operator Schmidt rank*. For a non-zero operator $X \in \mathcal{M}_n(\mathbb{C}) \otimes \mathcal{M}_m(\mathbb{C})$ the operator Schmidt rank [2, Lemma 10.1] is defined as the unique number $\Omega(X) \in \mathbb{N}$ such that

$$X = \sum_{i=1}^{\Omega(X)} A_i \otimes B_i \quad (1)$$

with orthogonal¹ sets of non-zero operators $\{A_i\}_{i=1}^r \subset \mathcal{M}_n(\mathbb{C})$ and $\{B_i\}_{i=1}^r \subset \mathcal{M}_m(\mathbb{C})$. Note that $\Omega(X) \in \{1, \dots, \min(n, m)^2\}$, and for general operators X all these ranks can occur. In this article, we are interested in the case of unitary operators X .

The question addressed in this work comes from the peculiar fact that the possible values of the Schmidt rank of unitary operators in dimensions $n = m = 2$ are restricted:

Theorem 1.1 ([10]). *For $U \in \mathcal{U}(\mathbb{C}^2 \otimes \mathbb{C}^2)$ only ranks $\Omega(U) \in \{1, 2, 4\}$ are possible.*

The result above can be proven either using the the so-called *normal form* of two-qubit unitary operations from [15], or using basic linear algebra [5, Theorem 3]. In [18] further restrictions on the

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¹with respect to the Hilbert-Schmidt inner product $\langle X, Y \rangle = \text{tr}(X^*Y)$ for $X, Y \in \mathcal{M}_n(\mathbb{C})$

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