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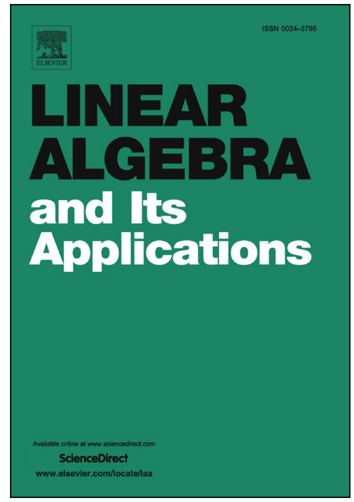

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Restrictions on the Schmidt rank of bipartite unitary operators beyond dimension two

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Abstract

There are none.

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1. Introduction

In the theory of quantum information [2], several notions of rank have been studied. The most prominent such notion is the Schmidt rank of pure bipartite quantum states, which plays an important role in entanglement theory [13, Section VI-A]. In this work, we study the possible values of a similar quantity for bipartite operators, the *operator Schmidt rank*. For a non-zero operator $X \in \mathcal{M}_n(\mathbb{C}) \otimes \mathcal{M}_m(\mathbb{C})$ the operator Schmidt rank [2, Lemma 10.1] is defined as the unique number $\Omega(X) \in \mathbb{N}$ such that

$$X = \sum_{i=1}^{\Omega(X)} A_i \otimes B_i \tag{1}$$

with orthogonal[1] sets of non-zero operators $\{A_i\}_{i=1}^r \subset \mathcal{M}_n(\mathbb{C})$ and $\{B_i\}_{i=1}^r \subset \mathcal{M}_m(\mathbb{C})$. Note that $\Omega(X) \in \left\{1, \ldots, \min(n,m)^2\right\}$, and for general operators X all these ranks can occur. In this article, we are interested in the case of unitary operators X.

The question addressed in this work comes from the peculiar fact that the possible values of the Schmidt rank of unitary operators in dimensions $n = m = 2$ are restricted:

Theorem 1.1 ([10]). *For $U \in \mathcal{U}\left(\mathbb{C}^2 \otimes \mathbb{C}^2\right)$ only ranks $\Omega(U) \in \{1, 2, 4\}$ are possible.*

The result above can be proven either using the the so-called *normal form* of two-qubit unitary operations from [15], or using basic linear algebra [5, Theorem 3]. In [18] further restrictions on the

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[1] with respect to the Hilbert-Schmidt inner product $\langle X, Y\rangle = \mathrm{tr}\,(X^* Y)$ for $X, Y \in \mathcal{M}_n(\mathbb{C})$