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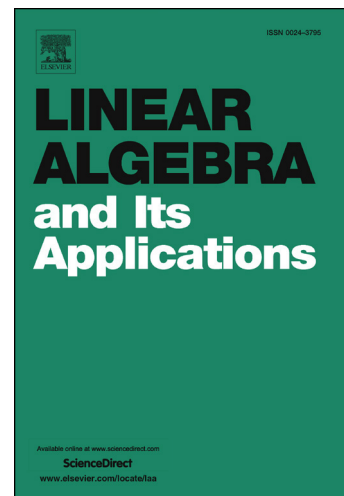
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Reduced commutativity of moduli of operators

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ABSTRACT. In this paper, we investigate the question of when the equations $A^{*s}A^s = (A^*A)^s$, $s \in S$, where S is a finite set of positive integers, imply the quasinormality or normality of A . In particular, it is proved that if $S = \{p, m, m+p, n, n+p\}$, where $p \geq 1$ and $2 \leq m < n$, then A is quasinormal. Moreover, if A is invertible and $S = \{m, n, n+m\}$ with $m \leq n$, then A is normal. The case when $S = \{m, m+n\}$ and $A^{*n}A^n \leq (A^*A)^n$ is also discussed.

1. Introduction

The class of bounded quasinormal operators was introduced by A. Brown in [5]. A bounded operator A on a (complex) Hilbert space \mathcal{H} is said to be *quasinormal* if $A(A^*A) = (A^*A)A$. Two different definitions of unbounded quasinormal operators appeared independently in [39] and in [49]. As recently shown in [35, Theorem 3.1], these two definitions are equivalent. Following [49, 154 pp.], we say that a closed densely defined operator A in \mathcal{H} is *quasinormal* if A commutes with the spectral measure E of $|A|$, i.e. $E(\sigma)A \subset AE(\sigma)$ for all Borel subsets σ of the nonnegative part of the real line. By [49, Proposition 1], a closed densely defined operator A in \mathcal{H} is quasinormal if and only if $U|A| \subset |A|U$, where $A = U|A|$ is the polar decomposition of A (see [53, Theorem 7.20]). For more information on quasinormal operators we refer the reader to [5, 15, 51] for the bounded case, and to [39, 49, 42, 35, 8, 51] for the unbounded one.

In 1973 M. R. Embry published a very influential paper [19] concerning the Halmos-Bram criterion for subnormality. In particular, she gave a characterisation of the class of quasinormal operators in terms of powers of operators. Namely, a bounded operator A in a Hilbert space is quasinormal if and only if the following condition holds

$$(1.1) \quad A^{*n}A^n = (A^*A)^n, \quad n \in \mathbb{N},$$

where \mathbb{N} stands for the set of all positive integers. This leads to the following question: is it necessary to assume that the equality in (1.1) holds for all $n \in \mathbb{N}$? To be more precise we ask for which subset $S \subset \mathbb{N}$ the following system of operator

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