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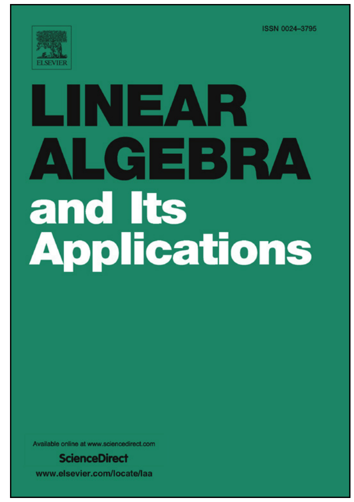

Reduced commutativity of moduli of operators

Paweł Pietrzycki

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Reduced commutativity of moduli of operators

Paweł Pietrzycki

Abstract. In this paper, we investigate the question of when the equations $A^{*s}A^{s} = (A^*A)^s$, $s \in S$, where S is a finite set of positive integers, imply the quasinormality or normality of A. In particular, it is proved that if $S = \{p, m, m+p, n, n+p\}$, where $p \geqslant 1$ and $2 \leqslant m < n$, then A is quasinormal. Moreover, if A is invertible and $S = \{m, n, n+m\}$ with $m \leqslant n$, then A is normal. The case when $S = \{m, m+n\}$ and $A^{*n}A^n \leqslant (A^*A)^n$ is also discussed.

1. Introduction

The class of bounded quasinormal operators was introduced by A. Brown in [**5**]. A bounded operator A on a (complex) Hilbert space $\mathcal{H}$ is said to be *quasinormal* if $A(A^*A) = (A^*A)A$. Two different definitions of unbounded quasinormal operators appeared independently in [**39**] and in [**49**]. As recently shown in [**35**, Theorem 3.1], these two definitions are equivalent. Following [**49**, 154 pp.], we say that a closed densely defined operator A in $\mathcal{H}$ is *quasinormal* if A commutes with the spectral measure E of $|A|$, i.e. $E(\sigma)A \subset AE(\sigma)$ for all Borel subsets σ of the nonnegative part of the real line. By [**49**, Proposition 1], a closed densely defined operator A in $\mathcal{H}$ is quasinormal if and only if $U|A| \subset |A|U$, where $A = U|A|$ is the polar decomposition of A (see [**53**, Theorem 7.20]). For more information on quasinormal operators we refer the reader to [**5, 15, 51**] for the bounded case, and to [**39, 49, 42, 35, 8, 51**] for the unbounded one.

In 1973 M. R. Embry published a very influential paper [**19**] concerning the Halmos-Bram criterion for subnormality. In particular, she gave a characterisation of the class of quasinormal operators in terms of powers of operators. Namely, a bounded operator A in a Hilbert space is quasinormal if and only if the following condition holds

$$A^{*n}A^n = (A^*A)^n, \quad n \in \mathbb{N}, \tag{1.1}$$

where $\mathbb{N}$ stands for the set of all positive integers. This leads to the following question: is it necessary to assume that the equality in (1.1) holds for all $n \in \mathbb{N}$? To be more precise we ask for which subset $S \subset \mathbb{N}$ the following system of operator

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