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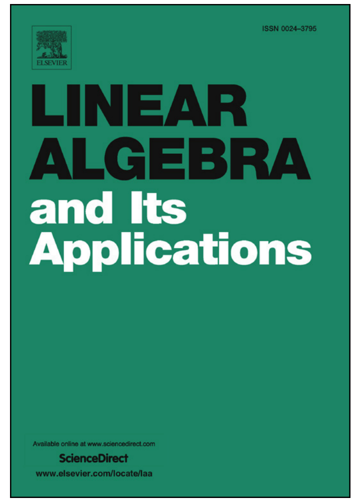

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A note on Möbius algebras and applications

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Abstract

We show a diagonalisation variant of Lindström calculation method. As an application of this result we calculate the dimension of the affine space of Negami's splitting matrices. We do so by writing down an expression for the Möbius function in term of Möbius algebra identities. As a corollary we get Lindström's result in a self contained way. Finally, we calculate the partition lattice characteristic polynomial via the Negami's polynomial.

Keywords: Möbius algebra, Möbius function, Negami's polynomial
2010 MSC: 15A54, 15A09, 15A21, 06B99, 05C31

1. Introduction

Let $\Gamma = \{x_1, x_2, \ldots x_n\}$ be a meet-semilattice and consider functions $f_i(x)$, $x \leq x_i$, with values in a commutative ring with unit. Define the matrices $N := (f_i(x_i \wedge x_j))$ and $M := (\mu(x_i, x_j))$ where μ is the Möbius function.

In [Li], it was shown that $NM = (a_{ij})$ is a triangular matrix whose diagonal elements are:

$$a_{ii} = \sum_{k=1}^{n} f_i(x_k)\mu(x_k, x_i)$$

In particular, the product of these elements gives the Lindström determinant:

$$\det\left(f_i(x_i \wedge x_j)\right) = \prod_{i=1}^{n} \sum_{j=1}^{n} f_i(x_j)\mu(x_j, x_i)$$

Now consider a single function $f(x)$, with values in a commutative ring with unit and define f_i as the restriction of f on the segment $[\mathbf{0}, x_i]$. In this paper