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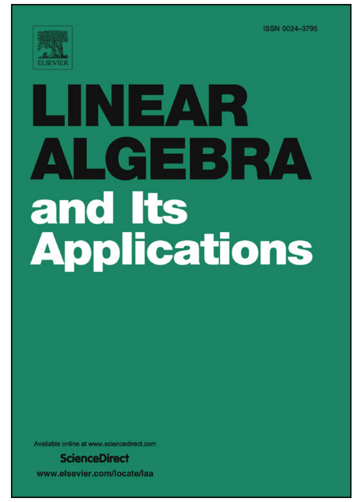

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ON JOINT SPECTRAL RADIUS OF COMMUTING OPERATORS IN HILBERT SPACES

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ABSTRACT. Our aim in this paper is to give a new formula of the joint spectral radius of commuting d-tuples of operators on a complex Hilbert space. Also we show that $r(\mathbf{T}) \leq \omega(\mathbf{T})$ for a commuting d-tuple of operators $\mathbf{T}$, where $r(\mathbf{T})$ and $\omega(\mathbf{T})$ denote respectively the joint spectral radius and the joint numerical radius of $\mathbf{T}$. This generalizes the well known relation between the spectral and the numerical radii of Hilbert space operators which is proved in [10].

1. INTRODUCTION

In the sequel, $\mathcal{H}$ is a complex Hilbert space, with inner product $\langle \cdot \mid \cdot \rangle$ and norm $\|\cdot\|$. The Banach algebra of all bounded linear operators on $\mathcal{H}$ will be denoted by $\mathcal{B}(\mathcal{H})$. In all that follows, by an operator we mean a bounded linear operator.

Let $\mathbf{T} = (T_1, \cdots, T_d) \in \mathcal{B}(\mathcal{H})^d$ be a d-tuple of commuting operators (i.e. the operators T_k are pairwise commuting). There are several different notions of a spectrum. For a good description, the reader is referred to [8] and the references there. There is a well-known notion of a joint spectrum of a commuting d-tuple $\mathbf{T}$ called the Taylor joint spectrum [14]. This is a compact subset of $\mathbb{C}^d$ and will be denoted here by the symbol $\sigma_T(\mathbf{T})$. The joint spectral radius of $\mathbf{T}$ is defined to be the number

$$r(\mathbf{T}) = \max\{\|\lambda\|_2,\ \lambda = (\lambda_1, \cdots, \lambda_d) \in \sigma_T(\mathbf{T})\}, \tag{1.1}$$

where $\|\lambda\|_2$ is the euclidean norm of $\lambda \in \mathbb{C}^d$. Notice that Chō and Żelazko proved in [7] that this definition of $r(\mathbf{T})$ is independent of the choice of the joint spectrum of $\mathbf{T}$. In particular one can replace $\sigma_T(\mathbf{T})$ by another joint spectrum of $\mathbf{T}$ in (1.1) without changing the value of $r(\mathbf{T})$.

Furthermore, an analogue of the Gelfand-Beurling spectral radius formula for single operators has been established by Müller, and Soltysiak in [12] for commuting tuples. More precisely, the joint spectral radius of a commuting d-tuple of operators

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