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Linearizations of matrix polynomials in Newton bases

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ABSTRACT

We discuss matrix polynomials expressed in a Newton basis, and the associated polynomial eigenvalue problems. Properties of the generalized ansatz spaces associated with such polynomials are proved directly by utilizing a novel representation of pencils in these spaces. Also, we show how the family of Fiedler pencils can be adapted to matrix polynomials expressed in a Newton basis. These new Newton–Fiedler pencils are shown to be strong linearizations, and some computational aspects related to them are discussed.

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1. Introduction

Consider a nonlinear eigenvalue problem $P(\lambda)x = 0, x \neq 0$, where $P(\lambda)$ is a matrix polynomial of the form

$$P(\lambda) = \sum_{i=0}^{k} A_i \phi_i(\lambda)\,, \quad A_0, A_1, \dots A_k \in \mathbb{F}^{n\times n}\,. \tag{1.1}$$

Here $\mathcal{B} = \{\phi_i(\lambda)\}_{i=0}^{k}$ is a polynomial basis for the space of univariate scalar polynomials of degree at most k; classical examples of such bases include Chebyshev, Newton, Hermite, Lagrange, Bernstein, etc. Matrix polynomials expressed in those bases arise either directly from applications, or as approximations when solving more general nonlinear eigenvalue problems, see for example [7,15,20,36,37] and the references therein.

The classical and most widely used approach to solving the polynomial eigenvalue problem $P(\lambda)x = 0$ is to first *linearize* $P(\lambda)$, i.e., convert $P(\lambda)$ into a matrix pencil $L(\lambda)$ with the same spectral structure, and then compute with $L(\lambda)$. Since the 1950's this approach has been extensively developed for matrix polynomials expressed in the *standard* basis. It was not until recently, though, that polynomial eigenvalue problems expressed in other bases have been seriously considered, see for example [1,5,15,29,32–37]. On the one hand, it is tempting to simply convert $P(\lambda)$ from (1.1) to the standard basis, and then leverage the existing body of knowledge about linearizations. However, it is important to avoid reformulating $P(\lambda)$ into the standard basis, since a change of basis has the potential to introduce numerical errors not present in the original statement of the problem.

In this paper we focus on *matrix polynomials expressed in a Newton basis*, i.e., polynomials of the form

$$P(\lambda) = \sum_{i=0}^{k} A_i n_i(\lambda)\,, \quad A_0, A_1, \dots A_k \in \mathbb{F}^{n\times n}\,, \tag{1.2}$$

where $n_i(\lambda)$ is the ith degree Newton polynomial associated with an ordered list of nodes $(\alpha_1, \dots, \alpha_k)$. Our main goal is to generate large new families of linearizations for $P(\lambda)$ by working *directly* with the coefficients A_i from (1.2), while avoiding additions, subtractions, multiplications, or inverses of those coefficients.

The first effort to study strong linearizations for matrix polynomials expressed in a Newton basis was made in [1], where a single example generalizing the first Frobenius companion form was provided. More recent results that extend the work from [22,26] consider the spaces of potential linearizations for *regular* matrix polynomials expressed in arbitrary degree-graded polynomial bases [32] and for *regular and singular* matrix polynomials expressed in orthogonal polynomial bases [16].

In this work we also aim to generalize the results of [26], but in contrast to [32] where such generalizations are obtained via a bivariate polynomial approach with emphasis