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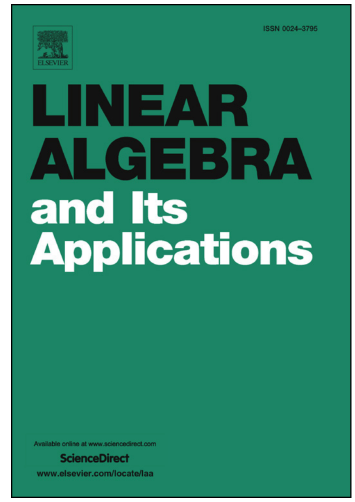
Michal Parnas, Adi Shraibman



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The Augmentation Property of Binary Matrices for the Binary and Boolean Rank

Michal Parnas
The Academic College
of Tel-Aviv-Yaffo
Tel-Aviv, Israel
michalp@mta.ac.il

Adi Shraibman
The Academic College
of Tel-Aviv-Yaffo
Tel-Aviv, Israel
adish@mta.ac.il

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Abstract

We define the Augmentation property for binary matrices with respect to different rank functions. A matrix A has the Augmentation property for a given rank function, if for any subset of column vectors $x_1, ..., x_t$ for which the rank of A does not increase when augmented separately with each of the vectors x_i, $1 \leq i \leq t$, it also holds that the rank does not increase when augmenting A with all vectors $x_1, ..., x_t$ simultaneously. This property holds trivially for the usual linear rank over the reals, but as we show, things change significantly when considering the binary and boolean rank of a matrix.

We prove a necessary and sufficient condition for this property to hold under the binary and boolean rank of binary matrices. Namely, a matrix has the Augmentation property for these rank functions if and only if it has a unique base that spans all other bases of the matrix with respect to the given rank function. For the binary rank, we also present a concrete sufficient characterization of a family of matrices that has the Augmentation property. This characterization is based on the possible types of linear dependencies between rows of V, in optimal binary decompositions of the matrix as $A = U \cdot V$.

Key Words— Communication Complexity, Binary rank, Boolean Rank
Classification Codes— 68Q99, 05C50, 15B34, 15B99

1 Introduction

The notion of the rank of a matrix over $\mathbb{R}$, or the rank over any field for that matter, is well understood. Many powerful techniques, most of which come from linear algebra, were devised over the years, which enable us to prove strong results involving the real rank of a matrix. For this reason, the rank appears in numerous situations in different areas of mathematics, including the field of theoretical computer science. In Section 1.1 we illustrate this by elaborating on the role of the rank function in the field of communication complexity, a role which started with a lower bound of Melhorn and Schmidt [15]. As in communication complexity, the linear algebraic tools and the many properties of the rank function, are those that make it attractive to use in many applications.

Denote by $R_{\mathbb{R}}(A)$ the rank of a real $n \times m$ matrix A over $\mathbb{R}$. Naming just a few of the useful properties of $R_{\mathbb{R}}(A)$, we have:

- $R_{\mathbb{R}}(A)$ is equal to the minimal size of a spanning set among the rows/columns of A.