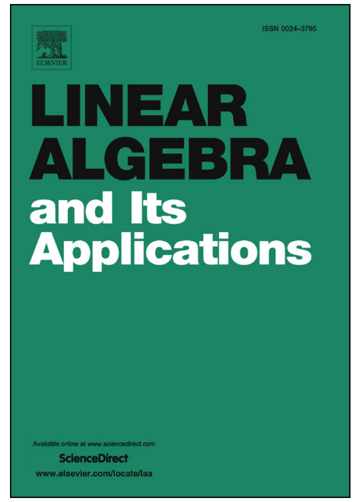
Accepted Manuscript



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PII: S0024-3795(18)30317-3
DOI: https://doi.org/10.1016/j.laa.2018.06.033
Reference: LAA 14641

To appear in: *Linear Algebra and its Applications*

Received date: 16 October 2017
Accepted date: 30 June 2018

Please cite this article in press as: K. Lentzos, L.F. Pasley, Determinantal representations of invariant hyperbolic plane curves, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2018.06.033

Determinantal representations of invariant hyperbolic plane curves

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Abstract

We study hyperbolic polynomials with nice symmetry and express them as the determinant of a Hermitian matrix with special structure. The goal of this paper is to answer a question posed by Chien and Nakazato in 2015. By properly modifying a determinantal representation construction of Dixon (1902), we show for every hyperbolic polynomial of degree n invariant under the cyclic group of order n there exists a determinantal representation admitted via some cyclic weighted shift matrix. Moreover, if the polynomial is invariant under the action of the dihedral group of order n, the associated cyclic weighted shift matrix is unitarily equivalent to one with real entries.

Keywords: Determinantal representation, Hyperbolic polynomial, Numerical range, Invariant curve, Weighted shift matrix, Dihedral invariance
2010 MSC: 14H50, 15A60, 14P99

1. Introduction

Let f be a real homogeneous polynomial of degree n in three variables t, x, y, so $\mathcal{V}_{\mathbb{C}}(f)$ is a projective plane curve. A determinantal representation of f is an expression

$$f = \det(tM_0 + xM_1 + yM_2),$$

where M_0, M_1, M_2 are $n \times n$ matrices. We set $M = M(t, x, y) = tM_0 + xM_1 + yM_2$ and refer to M as the determinantal representation of f. The representation is called real symmetric or Hermitian if M is of the respective form. Real symmetric and Hermitian determinantal representations have been systematically studied by Dubrovin [5] and Vinnikov [18, 19] in the late 1980's and early 1990's. Definite Hermitian determinantal representations are those for which there exists a point $e = (e_0, e_1, e_2) \in \mathbb{R}^3$ such that the matrix $M(e) = e_0M_0 + e_1M_1 + M_2e_2$ is positive definite. Since the eigenvalues of a Hermitian matrix are real, every real line passing through e meets the hypersurface $\mathcal{V}_{\mathbb{C}}(f)$ in only real points. Polynomials with this property are called hyperbolic and are intimately linked with convex optimization, see for example [1], [7] and [14].

Definition 1.1. *A homogeneous polynomial $f \in \mathbb{R}[t, x, y]_n$ is called* hyperbolic with respect to a point $e \in \mathbb{R}^3$ *if $f(e) \neq 0$ and for every $z \in \mathbb{R}^3$, all roots of the univariate polynomial $f(e+\lambda z) \in \mathbb{R}[\lambda]$ are real.*

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