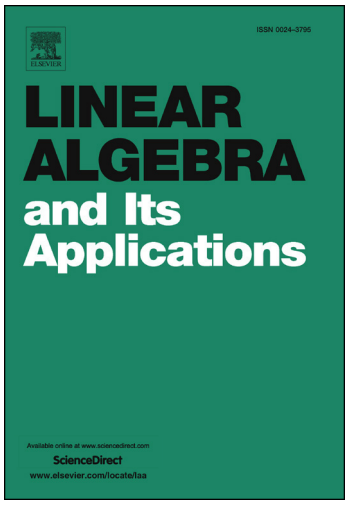

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Di Lu, Chun-Hua Guo

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Monotonicity and positivity of coefficients of power series expansions associated with Newton and Halley methods for the matrix pth root

Di Lu[a], Chun-Hua Guo[b]

[a] *Department of Mathematics and Statistics, University of Regina, Regina, Saskatchewan S4S 0A2, Canada*
[b] *Department of Mathematics and Statistics, University of Regina, Regina, Saskatchewan S4S 0A2, Canada*

Abstract

If A is a matrix with all eigenvalues in the disk $|z-1|<1$, the principal pth root of A can be computed by Newton's method or Halley's method. The study of Newton's method and Halley's method for the matrix pth root can be done through a study of power series expansions of some sequences of scalar functions. In this paper, we prove monotonicity results for the coefficients in the power series expansions for both Newton's method and Halley's method, and for all integer $p \geq 2$. The sign patterns of these coefficients can be seen directly from the monotonicity results. We then use these monotonicity results and their consequences to obtain some new error estimates in the matrix case, to obtain a monotonic convergence result when A is a nonsingular M-matrix, and to obtain a structure preserving result when A is a nonsingular M-matrix or a real nonsingular H-matrix with positive diagonal entries.

AMS classification: 15A16; 65F60

Keywords: Matrix pth root; Newton's method; Halley's method; Series expansion; M-matrix; H-matrix

1. Introduction

Let $p \geq 2$ be an integer. Suppose that $A \in \mathbb{C}^{n\times n}$ has all eigenvalues in the open disk $\{z \in \mathbb{C} : |z-1|<1\}$. Then the principal pth root of A, denoted by $A^{1/p}$, exists and can be computed by various methods [2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 18].

This paper is a continuation of [5] and is exclusively about Newton's method and Halley's method.

Email addresses: ludix203@uregina.ca (Di Lu), chun-hua.guo@uregina.ca (Chun-Hua Guo)