

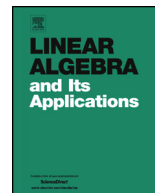


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Some spectral norm inequalities on Hadamard products of nonnegative matrices [☆]

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ARTICLE INFO

Article history:

Received 19 May 2018

Accepted 8 July 2018

Available online xxxx

Submitted by X. Zhan

MSC:

15A18

15A60

15A69

15B48

Keywords:

Hadamard product

Nonnegative matrices

Spectral radius

Spectral norm

ABSTRACT

Let A and B be nonnegative square matrices of the same order. Denote by $\|\cdot\|$ and $\rho(\cdot)$ the spectral norm and the spectral radius respectively. We prove the following inequalities:

$$\|A \circ B\| \leq \|A \circ A\|^{\frac{1}{2}} \|B \circ B\|^{\frac{1}{2}};$$

$$\|A \circ B\| \leq \rho^{\frac{1}{2}}(A^T B \circ B^T A) \leq \rho^{\frac{1}{2}}(A^T B \circ A^T B) \leq \rho(A^T B),$$

where \circ denotes the Hadamard product. This interpolates the inequality

$$\|A \circ B\| \leq \rho(A^T B)$$

due to Huang. Some spectral norm inequalities for an arbitrarily finite number of nonnegative square matrices are also obtained, which refine some other results of Huang.

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[☆] Supported by Anhui Provincial Natural Science Foundation (1708085QA05), The Key Program in the Youth Elite Support Plan in Universities of Anhui Province (gxyqZD2018047) and the Natural Science Foundation of Anhui Higher Education Institutions of China (KJ2016B001).

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1. Introduction and preliminaries

Let M_n denote the set of complex matrices of order n . For matrices $A = (a_{ij}), B = (b_{ij}) \in M_n$, we denote by $\rho(A)$ the spectral radius of A , by $A \circ B = (a_{ij}b_{ij})$ the Hadamard product of A and B . The notation $A \leq B$ means that $B - A$ is entrywise nonnegative. For $T \in M_n$, the *singular values* of T , denoted by $s_1(T) \geq s_2(T) \geq \dots \geq s_n(T)$, are the eigenvalues of the positive semidefinite matrix $|T| = (T^*T)^{\frac{1}{2}}$. It follows that the singular values of a normal matrix are just the moduli of its eigenvalues. Denote by $\|A\|$ the spectral norm of $A \in M_n$, which equals the largest singular value.

Zhan [11] conjectured that $\rho(A \circ B) \leq \rho(AB)$ for nonnegative matrices $A, B \in M_n$. This conjecture was confirmed by Audenaert in [1] by proving

$$\rho(A \circ B) \leq \rho^{\frac{1}{2}}((A \circ A)(B \circ B)) \leq \rho(AB). \tag{1.1}$$

Using the fact that the Hadamard product of two matrices is a principal submatrix of the Kronecker product, Horn and Zhang proved [4] the inequalities

$$\rho(A \circ B) \leq \rho^{\frac{1}{2}}(AB \circ BA) \leq \rho(AB). \tag{1.2}$$

Huang [5] generalized the inequality $\rho(A \circ B) \leq \rho(AB)$ to an arbitrarily finite number of nonnegative matrices:

$$\rho(A_1 \circ A_2 \circ \dots \circ A_k) \leq \rho(A_1 A_2 \dots A_k). \tag{1.3}$$

Over the years, various generalizations on the spectral radius of Hadamard products of nonnegative matrices have been considered in the literature; e.g., [2,6–10].

The aim of this paper is to present some inequalities on the spectral norm of the Hadamard product of nonnegative matrices. The main results are the following

- (1) Let $A, B \in M_n$ be nonnegative matrices. Then

$$\|A \circ B\| \leq \|A \circ A\|^{\frac{1}{2}} \|B \circ B\|^{\frac{1}{2}};$$

Denote by S_k the set of all permutations of $1, 2, \dots, k$.

- (2) Let $A_1, A_2, \dots, A_k \in M_n$ be nonnegative matrices. Then for any $\tau, \gamma \in S_k$

$$\|A_1 \circ A_2 \circ \dots \circ A_k\| \leq \rho^{\frac{1}{2}} \left(A_{\tau(1)} A_{\gamma(1)}^T \circ A_{\tau(2)} A_{\gamma(2)}^T \circ \dots \circ A_{\tau(k)} A_{\gamma(k)}^T \right).$$

If k is even, then for any $\tau \in S_k$,

$$\|A_1 \circ A_2 \circ \dots \circ A_k\| \leq \rho \left(A_{\tau(1)}^T A_{\tau(2)} \circ A_{\tau(3)}^T A_{\tau(4)} \circ \dots \circ A_{\tau(k-1)}^T A_{\tau(k)} \right)$$

By specifying suitable permutations in (2), it can generalize some useful inequalities, which can refine some results due to Huang [5]. As an application, it implies that for nonnegative matrices $A, B \in M_n$,

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