

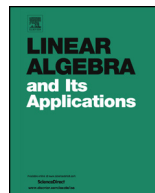


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Operator Ky Fan type inequalities



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ABSTRACT

In this paper, we extend some significant Ky Fan type inequalities in a large setting to operators on Hilbert spaces and derive their equality conditions. Among other things, we prove that if $f : [0, \infty) \rightarrow [0, \infty)$ is an operator monotone function with $f(1) = 1$, $f'(1) = \mu$, and associated mean σ , then for all operators A and B on a complex Hilbert space $\mathcal{H}$ such that $0 < A, B \leq \frac{1}{2}I$, we have

$$A' \nabla_{\mu} B' - A' \sigma B' \leq A \nabla_{\mu} B - A \sigma B,$$

where I is the identity operator on $\mathcal{H}$, $A' := I - A$, $B' := I - B$, and ∇_{μ} is the μ -weighted arithmetic mean.

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1. Introduction

Let $n \geq 2$, and let $\mu_1, \dots, \mu_n \geq 0$ such that $\sum_{i=1}^n \mu_i = 1$. For arbitrary real numbers $x_1, \dots, x_n > 0$, we denote by A_n, G_n , and H_n the arithmetic mean, the geometric mean, and the harmonic mean of $x_1, \dots, x_n$, respectively; that is,

$$A_n = \sum_{i=1}^n \mu_i x_i, \quad G_n = \prod_{i=1}^n x_i^{\mu_i}, \quad H_n = \frac{1}{\sum_{i=1}^n \mu_i \frac{1}{x_i}}.$$

For $x_i \in (0, \frac{1}{2}]$, we denote by A'_n, G'_n , and H'_n the arithmetic, geometric, and harmonic means of $x'_1 := 1 - x_1, \dots, x'_n := 1 - x_n$, respectively; that is,

$$A'_n = \sum_{i=1}^n \mu_i x'_i, \quad G'_n = \prod_{i=1}^n x_i'^{\mu_i}, \quad H'_n = \frac{1}{\sum_{i=1}^n \mu_i \frac{1}{x'_i}}.$$

The most important and elegant Ky Fan type inequalities can be divided in the following three classes:

- **Additive class:**

$$A'_n - G'_n \leq A_n - G_n, \quad (\text{Alzer}) \quad (1.1)$$

$$A'_n - H'_n \leq A_n - H_n. \quad (\text{Alzer}) \quad (1.2)$$

- **Reciprocal additive class:**

$$\frac{1}{G'_n} - \frac{1}{A'_n} \leq \frac{1}{G_n} - \frac{1}{A_n}, \quad (1.3)$$

$$\frac{1}{H'_n} - \frac{1}{G'_n} \leq \frac{1}{H_n} - \frac{1}{G_n}, \quad (1.4)$$

$$\frac{1}{H'_n} - \frac{1}{A'_n} \leq \frac{1}{H_n} - \frac{1}{A_n}. \quad (1.5)$$

- **Multiplicative class:**

$$\frac{A'_n}{G'_n} \leq \frac{A_n}{G_n}, \quad (\text{Ky Fan}) \quad (1.6)$$

$$\frac{G'_n}{H'_n} \leq \frac{G_n}{H_n}, \quad (\text{Wang-Wang}) \quad (1.7)$$

$$\frac{A'_n}{H'_n} \leq \frac{A_n}{H_n}. \quad (1.8)$$

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