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The exponent for superalgebras with superinvolution

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A B S T R A C T

Let A be a superalgebra with superinvolution over a field of characteristic zero and let $c_n^*(A)$, $n = 1, 2, \ldots$, be its sequence of $*$-codimensions. In [6] it was proved that such a sequence is exponentially bounded.

In this paper we capture this exponential growth for finitely generated superalgebras with superinvolution A over an algebraically closed field of characteristic zero. We shall prove that $\lim_{n\to\infty} \sqrt[n]{c_n^*(A)}$ exists and it is an integer, denoted $\exp^*(A)$ and called $*$-exponent of A. Moreover, we shall characterize finitely generated superalgebras with superinvolution according to their $*$-exponent.

1. Introduction

Let A be an associative algebra over a field F of characteristic zero and let $c_n(A)$ be its sequence of codimensions, introduced by Regev in 1972. In [25] he proved that, if A satisfies a non-trivial identity (PI-algebra), then such a sequence is exponentially

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bounded. Moreover, a famous result of Kemer (see [20,21]) stated that no intermediate growth of the codimensions between polynomial and exponential is allowed.

According to several examples provided by Berele and Regev, in the early 1980's, Amitsur conjectured that the exponential growth of the codimension sequence should be an integer. In 1999, Giambruno and Zaicev confirmed this conjecture by proving that the limit $\exp(A) := \lim_{n\to\infty} \sqrt[n]{c_n(A)}$, called exponent of the algebra A, exists and it is an integer. They gave an explicit way to compute the exponent of any PI-algebra ([11,12]) and in [14] characterized associative algebras of exponent 2.

From that moment, several authors studied the problem posed by Amitsur and, more generally, the exponential growth of the codimension sequence in the setting of algebras with some additional structure. Between them, we recall the cases of algebras with involution ([10,13,22]), superalgebras ([5]) and more generally algebras graded by a group ([1,3,9]), algebras with a generalised H-action ([16]) and superalgebras with graded involution ([26]).

In this paper we deal with superalgebras with superinvolution. They are a natural generalization of the algebras with involution and they play a prominent role in the setting of Lie and Jordan superalgebras (see [19,24]).

As in the ordinary case, one can attach to a superalgebra with superinvolution A the numerical sequence of $*$-codimensions $c_n^*(A)$. If A satisfies an ordinary non-trivial identity, then such a sequence is exponentially bounded (see [6]).

In the first part of this paper we shall investigate the exponential behaviour of the $*$-codimension sequence and we shall prove that, if A is a finitely generated superalgebras with superinvolution satisfying an ordinary non-trivial identity, then the limit $\lim_{n\to\infty} \sqrt[n]{c_n^*(A)}$ exists and it is an integer, denoted $\exp^*(A)$ and called $*$-exponent of A.

The last part of this paper is devoted to the study of finitely generated superalgebras with superinvolution according to their $*$-exponent. First we shall see that for an algebra A of this kind the condition of having the $*$-exponent $\exp^*(A) \leq 1$ means that A has polynomial growth, i.e., the codimension sequence $c_n^*(A)$, $n = 1, 2, \ldots$, is polynomially bounded. Next, we shall exhibit nine suitable finite dimensional superalgebras with superinvolution that will allow us to characterize those algebras with $*$-exponent greater than two. As immediate corollary, we get the characterization of the finitely generated superalgebras with superinvolution of exponent two.

2. Preliminaries and basic results

Let F be a field of characteristic zero and let $A = A_0 \oplus A_1$ be an associative superalgebra over F endowed with a superinvolution $*$. The subspaces A_0 and A_1 satisfy the conditions $A_0A_0 + A_1A_1 \subseteq A_0$ and $A_0A_1 + A_1A_0 \subseteq A_1$ and their elements are called homogeneous of degree zero (even elements) and of degree one (odd elements), respectively. A superinvolution on A is a graded linear map $* : A \to A$ such that $(x^*)^* = x$, for all $x \in A$, and $(ab)^* = (-1)^{|a||b|} b^*a^*$, for elements $a, b \in A$ of homogeneous degree