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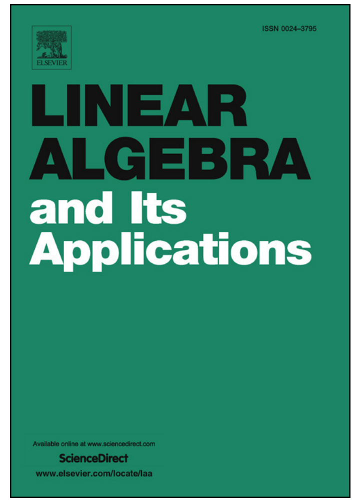

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On the sum of Laplacian eigenvalues of a signed graph

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Abstract

For a signed graph Γ, let $e(\Gamma)$ denote the number of edges and $S_k(\Gamma)$ denote the sum of the k largest eigenvalues of the Laplacian matrix of Γ. We conjecture that for any signed graph Γ with n vertices, $S_k(\Gamma) \leq e(\Gamma) + \binom{k+1}{2} + 1$ holds for $k = 1, \ldots, n$. We prove the conjecture for any signed graph when $k = 2$, and prove that this conjecture is true for unicyclic and bicyclic signed graphs.
MSC: 05C50
Keywords: signed graphs, sum of eigenvalues, Laplacian matrix

1 Introduction

In this paper, all graphs are simple (loopless and without multiple edges). The vertex set and edge set of the graph G will be denoted by $V(G)$ and $E(G)$, respectively. A *signed graph* (or sigraph for short) $\Gamma = (G, \sigma)$ consists of an unsigned graph $G = (V, E)$ and a sign function $\sigma : E(G) \to \{+, -\}$, and G is its underlying graph, while σ is its sign function (or signature). Furthermore, it is also to interpret the signs as the integers $\{+1, -1\}$. Hence, signed graphs sometimes are treated as weighted graphs, whose (edge) weights is 1 or -1. An edge e is *positive* (*negative*) if $\sigma(e) = +$ (resp. $\sigma(e) = -$). If all edges in Γ are *positive* (*negative*), then Γ is denoted by $(G, +)$ (resp. $(G, -)$).

Actually, each concept defined for the underlying graph can be transferred with signed graphs. For example, the degree of a vertex v in G is also its degree in Γ. Furthermore, if some subgraph of the underlying graph is observed, then the sign function for the subgraph is the restriction of the previous one. Thus, if $v \in V(G)$, then $\Gamma - v$ denotes the signed subgraph whose underlying graph is $G - v$ and its signature is the restriction from $E(G)$ to $E(G - v)$. If $U \subseteq V(G)$ then $\Gamma[U]$ or $G(U)$ denotes the (signed) induced subgraph arising from U, while $\Gamma - U = \Gamma[V(G)\backslash U]$. Sometimes we also write $\Gamma - \Gamma[U]$

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